

# Appendix A - Statistical Tables

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Table A.2.....Standard Normal Distribution

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Table A.4.....Fisher's  $f$  Distributions

Table A.5.....Chi-Square ( $\chi^2$ ) Distributions

# Table A.1

## Table of Random Numbers

52285 53301 71193 18991 34854 61701 10262 41876 19487 06996  
70189 45193 46899 90746 97060 46547 64523 16987 60706 51116  
83441 17072 50243 83300 63817 07510 05828 95271 07689 29757  
16254 51933 02155 93543 20033 88132 16695 58878 39877 32928  
84056 70489 14252 94132 04605 38293 60501 20415 82886 04396

01398 76800 59510 70789 59184 72725 81987 60820 67407 06777  
46083 69602 70703 54693 85747 69453 40158 84787 65193 07982  
74331 57826 36074 65258 56350 67475 10856 05061 54175 45490  
25271 57349 62441 93647 49612 26541 48268 02745 50788 47621  
39202 76311 95471 81348 27869 51539 78557 98949 37103 52406

34579 78142 36414 51185 76519 93762 52903 80133 50426 60660  
05815 46497 15942 97225 73766 07688 03995 63519 96185 79889  
12256 07450 14703 66624 65096 05991 47792 58688 42390 72273  
47813 55926 97304 99955 38725 88637 32076 00059 69312 80328  
14387 76015 22189 48236 40675 56748 08779 05052 81391 91175

12871 38800 45132 29899 05541 97211 00228 37943 75643 24707  
01790 16008 71773 70610 30776 99122 06371 95624 85352 10541  
51382 87353 27396 55549 61991 35378 10592 91612 24863 82825  
07971 05251 88080 17223 00214 02158 32327 39454 00488 12942  
10387 42632 71863 15885 19797 33809 43331 23947 52006 49206

12011 45094 27304 69411 93673 44863 57021 27073 02530 05554  
58606 04896 14194 50974 15115 47297 46172 09915 53970 45107  
32011 56324 53923 94211 02603 31294 56761 62518 28402 94369  
76525 54749 54816 27929 16543 25163 23840 82143 67689 79686  
00885 78999 80525 67499 38229 07047 70281 27868 28125 70575

23897 16928 42382 97327 00010 12017 09077 55481 11533 67540  
48352 74755 54420 95487 88860 15474 71007 71176 15957 55571  
84380 89850 63181 12825 39798 64614 75342 59859 67318 25476  
66567 06900 85623 22070 69356 51678 13675 05150 96915 45290  
69947 95093 39012 02347 65222 14159 89423 39265 32780 51071

32141 18317 80061 61293 46152 44829 58578 36048 44759 64105  
45368 48058 18536 63557 56784 04185 01216 22678 62302 24258  
52706 41960 83558 74152 64791 79863 70243 23171 27735 78682  
24526 79798 65401 66916 12948 49739 83458 02173 36878 75965  
11143 51658 61071 92410 67354 71470 50351 49670 90852 53216

90436 16823 71004 28796 31809 88289 64149 41630 10620 71445  
50695 16631 30928 17151 48671 89919 33993 24160 00432 22646  
12661 40143 80449 48565 66144 02950 23605 14699 88568 94758  
26095 37076 50570 47774 35325 53309 57110 81659 44003 47856  
37401 08878 38040 40886 79776 17453 16386 21934 51879 98593

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# Table A.1

## Table of Random Numbers

*(Continued From Previous Page)*

73436 15235 68742 37408 54480 14276 19199 09247 51883 31558  
52085 88482 90978 50213 81692 71534 55172 89050 75064 22917  
74921 67644 43406 47325 74620 57386 21548 45531 23754 57318  
11779 30845 04668 57815 86485 68987 65038 86689 37010 89742  
12697 40295 38712 00802 12178 04126 79675 89010 56424 10949

01530 95937 54384 88243 69773 47554 87366 84193 32006 74961  
85512 28259 62138 06750 74984 82732 39352 47103 66217 69987  
56593 44037 62860 83469 59660 03227 30197 38752 01730 91522  
48971 46910 05183 47636 38411 46134 05488 43152 50491 46963  
80174 37041 68187 14965 08437 09184 78120 77283 85683 03000

10451 16818 20909 41817 22258 05721 89967 52817 48289 37249  
15709 96644 07787 19769 89928 25008 75161 71145 87686 14697  
80410 73637 91873 19172 31133 03062 47472 77656 89860 29345  
78387 26525 83058 27609 39367 60404 17724 19381 09401 41116  
80340 45802 51136 53479 02947 21568 38732 80795 32185 67188

19303 67335 12415 52422 88966 42252 39225 12063 66612 96193  
65547 43596 26205 84974 19288 64531 30886 67894 59180 87874  
55138 63666 92563 73142 49480 87045 90442 51276 24575 08896  
41251 49346 92401 08983 37610 51640 32392 38966 94555 30858  
34360 33395 50573 54086 76272 04358 49068 74122 65103 46740

28798 62591 88671 40094 20671 94535 13831 73727 23013 17250  
28821 35325 49845 40713 10831 62889 31144 94793 47631 69475  
61406 11022 10801 82661 96007 82777 10886 50354 80586 36537  
53064 03232 18298 42549 17696 90115 73195 46877 43756 65747  
10340 07841 84279 96642 43454 32981 36294 62135 87647 37954

75471 44635 36918 78946 58286 46874 08289 02970 45582 97166  
54595 16847 31134 89115 09788 97384 69642 64739 60784 08725  
46054 66831 76812 76767 33350 66654 32282 46201 38030 61321  
64056 31307 94018 92901 18269 76377 77698 36684 01007 31710  
48772 39634 51600 09518 90956 87022 30606 24204 42723 99132

71878 37326 75740 56392 33145 48232 04240 85284 24372 70326  
52795 28840 07950 09409 59846 58692 84039 66761 14916 74160  
94307 80909 98649 46434 65594 18673 27853 77889 54909 49947  
36496 61287 09743 69322 00658 57232 68305 88356 10208 65712  
93837 94788 28566 25575 69803 02395 80901 40244 25023 58347

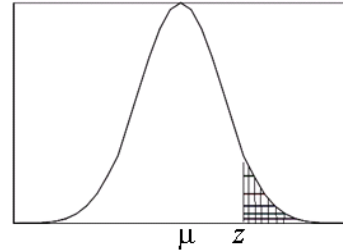
62769 19152 42725 06747 32435 50598 47708 66061 26076 77413  
01441 77154 23681 26553 06565 60362 97591 65225 55668 47806  
52357 67042 87617 05415 84880 38953 67029 58816 03215 41258  
91948 81731 28846 88081 38023 26118 25129 69856 67321 65109  
49574 96113 07275 51855 73484 97206 38430 93330 87042 50463

## Table A.2

### Standard Normal Distribution

Areas reported below correspond to the shaded region in the figure on the right.

$$z = \frac{x - \mu}{\sigma}$$



$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010

$$z_{0.10} = 1.282$$

$$z_{0.01} = 2.326$$

$$z_{0.05} = 1.645$$

$$z_{0.005} = 2.576$$

$$z_{0.025} = 1.960$$

$$z_{0.0005} = 3.291$$

## Table A.3

### Student's $t$ Distributions

Rows are labeled by  $df$  (degrees of freedom).

Columns are labeled by tail probabilities of the type corresponding to the shaded region in the figure on the right.

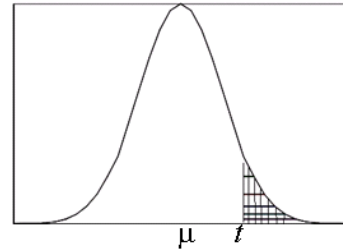


Table entries are  $t$  values.

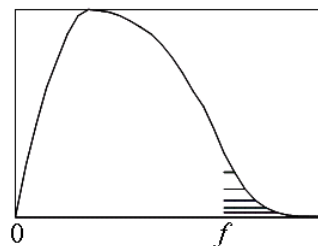
$df$	0.10	0.05	0.025	0.01	0.005	0.0005
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.921
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.595
7	1.415	1.895	2.365	2.998	3.499	5.405
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.767
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
$\infty$	1.282	1.645	1.960	2.326	2.576	3.291

## Table A.4 Fisher's $f$ Distributions

First, second, third, and fourth rows correspond respectively to the upper tail probabilities 0.10, 0.05, 0.01, 0.001.

Columns are labeled by tail probabilities of the type corresponding to the shaded region in the figure on the right.

Table entries are  $f$  values.



		<b>N u m e r a t o r   D e g r e e s   o f   F r e e d o m</b>									
		1	2	3	4	5	6	7	8	9	10
<b>D e n o m i n a t o r</b>	1	39.9 161 4052	49.5 200 5000	53.6 216 5403	55.8 225 5625	57.2 230 5764	58.2 234 5859	58.9 237 5928	59.4 239 5981	59.9 241 6022	60.2 242 6056
	2	8.53 18.5 98.5 999	9.00 19.0 99.0 999	9.16 19.2 99.2 999	9.24 19.2 99.2 999	9.29 19.3 99.2 999	9.33 19.3 99.3 999	9.35 19.4 99.3 999	9.37 19.4 99.4 999	9.38 19.4 99.4 999	9.39 19.4 99.4 999
	3	5.54 10.1 34.1 167	5.46 9.55 30.8 148	5.39 9.28 29.5 141	5.34 9.12 28.7 137	5.31 9.01 28.2 135	5.28 8.94 27.9 133	5.27 8.89 27.7 132	5.25 8.85 27.5 131	5.24 8.81 27.3 130	5.23 8.79 27.2 129
	4	4.54 7.71 21.2 74.1	4.32 6.94 18.0 61.2	4.19 6.59 16.7 56.2	4.11 6.39 16.0 53.4	4.05 6.26 15.5 51.7	4.01 6.16 15.2 50.5	3.98 6.09 15.0 49.7	3.95 6.04 14.8 49.0	3.94 6.00 14.7 48.5	3.92 5.96 14.5 48.1
<b>F r e e d o m</b>	5	4.06 6.61 16.3 47.2	3.78 5.79 13.3 37.1	3.62 5.41 12.1 33.2	3.52 5.19 11.4 31.1	3.45 5.05 11.0 29.8	3.40 4.95 10.7 28.8	3.37 4.88 10.5 28.2	3.34 4.82 10.3 27.6	3.32 4.77 10.2 27.2	3.30 4.74 10.1 26.9
	6	3.78 5.99 13.7 35.5	3.46 5.14 10.9 27.0	3.29 4.76 9.78 23.7	3.18 4.53 9.15 21.9	3.11 4.39 8.75 20.8	3.05 4.28 8.47 20.0	3.01 4.21 8.26 19.5	2.98 4.15 8.10 19.0	2.96 4.10 7.98 18.7	2.94 4.06 7.87 18.4
	7	3.59 5.59 12.2 29.2	3.26 4.74 9.55 21.7	3.07 4.35 8.45 18.8	2.96 4.12 7.85 17.2	2.88 3.97 7.46 16.2	2.83 3.87 7.19 15.5	2.78 3.79 6.99 15.0	2.75 3.73 6.84 14.6	2.72 3.68 6.72 14.3	2.70 3.64 6.62 14.1
	8										

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# Table A.4

## Fisher's $f$ Distributions

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First, second, third, and fourth rows correspond respectively to the upper tail probabilities 0.10, 0.05, 0.01, 0.001.

Columns are labeled by tail probabilities of the type corresponding to the shaded region in the figure on the right.

Table entries are  $f$  values.

		<b>N u m e r a t o r   D e g r e e s   o f   F r e e d o m</b>									
		1	2	3	4	5	6	7	8	9	10
<b>D e n o m i n a t o r</b>	8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54
		5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
		11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
		25.4	18.5	15.8	14.4	13.5	12.9	12.4	12.0	11.8	11.5
<b>D e g r e e s</b>	9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42
		5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
		10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
		21.0	16.4	12.6	11.3	10.5	9.93	9.52	9.20	8.96	8.75
<b>D e g r e e s</b>	10	3.92	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32
		4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
		10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
		21.0	14.9	12.6	11.3	10.5	9.93	9.52	9.20	8.96	8.75
<b>F r e e d o m</b>	11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25
		4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
		9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
		19.7	13.8	11.6	10.3	9.58	9.05	8.66	8.35	8.12	7.92
<b>F r e e d o m</b>	12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19
		4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
		9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
		18.6	13.0	10.8	9.63	8.89	8.38	8.00	7.71	7.48	7.29
<b>F r e e d o m</b>	13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14
		4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
		9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
		17.8	12.3	10.2	9.07	8.35	7.86	7.49	7.21	6.98	6.80
<b>F r e e d o m</b>	14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10
		4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
		8.88	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
		17.1	11.8	9.73	8.62	7.92	7.44	7.08	6.80	6.58	6.40

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# Table A.4

## Fisher's $f$ Distributions

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First, second, third, and fourth rows correspond respectively to the upper tail probabilities 0.10, 0.05, 0.01, 0.001.

Columns are labeled by tail probabilities of the type corresponding to the shaded region in the figure on the right.

Table entries are  $f$  values.

		<b>N u m e r a t o r   D e g r e e s   o f   F r e e d o m</b>									
		1	2	3	4	5	6	7	8	9	10
<b>D e n o m i n a t o r</b>	15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06
		4.54	3.68	3.29	3.05	2.90	2.79	2.71	2.64	2.59	2.54
		8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
		16.6	11.3	9.34	8.25	7.57	7.09	6.74	6.47	6.26	6.08
<b>D e g r e e s</b>	16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03
		4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
		8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
		16.1	11.0	9.01	7.94	7.27	6.80	6.46	6.19	5.98	5.81
<b>D e g r e e s</b>	17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00
		4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
		8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59
		15.7	10.7	8.73	7.68	7.02	6.56	6.22	5.96	5.75	5.58
<b>D e g r e e s</b>	18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98
		4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
		8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
		15.4	10.4	8.49	7.46	6.81	6.35	6.02	5.76	5.56	5.39
<b>D e g r e e s</b>	19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96
		4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
		8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
		15.1	10.2	8.28	7.27	6.62	6.18	5.85	5.59	5.39	5.22
<b>D e g r e e s</b>	20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94
		4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
		8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
		14.8	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	5.08
<b>D e g r e e s</b>	21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92
		4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
		8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
		14.6	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11	4.95

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# Table A.4

## Fisher's $f$ Distributions

(Continued From Previous Page)

First, second, third, and fourth rows correspond respectively to the upper tail probabilities 0.10, 0.05, 0.01, 0.001.

Columns are labeled by tail probabilities of the type corresponding to the shaded region in the figure on the right.

Table entries are  $f$  values.

		<b>N u m e r a t o r   D e g r e e s   o f   F r e e d o m</b>									
		1	2	3	4	5	6	7	8	9	10
<b>D e n o m i n a t o r</b>	22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90
		4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
		7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
		14.4	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	4.83
<b>D e g r e e s</b>	23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89
		4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
		7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
		14.2	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89	4.73
<b>D e g r e e s</b>	24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88
		4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
		7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
		14.0	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	4.64
<b>D e g r e e s</b>	25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87
		4.24	3.39	2.99	2.75	2.60	2.49	2.40	2.34	2.28	2.24
		7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13
		13.9	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71	4.56
<b>D e g r e e s</b>	26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86
		4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
		7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
		13.7	9.12	7.36	6.41	5.80	5.38	5.07	4.38	4.64	4.48
<b>D e g r e e s</b>	27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85
		4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
		7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
		13.6	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57	4.41
<b>D e g r e e s</b>	28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84
		4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
		7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
		13.5	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50	4.35

(Continued Next Page →)

# Table A.4

## Fisher's $f$ Distributions

(Continued From Previous Page)

First, second, third, and fourth rows correspond respectively to the upper tail probabilities 0.10, 0.05, 0.01, 0.001.

Columns are labeled by tail probabilities of the type corresponding to the shaded region in the figure on the right.

Table entries are  $f$  values.

		<b>N u m e r a t o r   D e g r e e s   o f   F r e e d o m</b>									
		1	2	3	4	5	6	7	8	9	10
<b>D e n o m i n a t o r</b>	29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83
		4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
		7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
		13.4	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45	4.29
<b>D e g r e e s</b>	30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82
		4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
		7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
		13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	4.24
<b>D e g r e e s</b>	32	2.87	2.48	2.26	2.13	2.04	1.97	1.91	1.87	1.83	1.81
		4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14
		7.50	5.34	4.46	3.97	3.65	3.43	3.26	3.13	3.02	2.93
		13.1	8.64	6.94	6.01	5.43	5.02	4.72	4.48	4.30	4.14
<b>D e g r e e s</b>	34	2.86	2.47	2.25	2.12	2.02	1.96	1.90	1.86	1.82	1.79
		4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12
		7.44	5.29	4.42	3.93	3.61	3.39	3.22	3.09	2.98	2.89
		13.0	8.52	6.83	5.92	5.34	4.93	4.63	4.40	4.22	4.06
<b>D e g r e e s</b>	36	2.85	2.46	2.24	2.11	2.01	1.94	1.89	1.85	1.81	1.78
		4.11	3.26	2.87	2.63	2.48	2.36	2.28	2.21	2.15	2.11
		7.40	5.25	4.38	3.89	3.57	3.35	3.18	3.05	2.95	2.86
		12.8	8.42	6.74	5.84	5.26	4.86	4.56	4.33	4.14	3.99
<b>D e g r e e s</b>	38	2.84	2.45	2.23	2.10	2.01	1.94	1.88	1.84	1.80	1.77
		4.10	3.24	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09
		7.35	5.21	4.34	3.86	3.54	3.32	3.15	3.02	2.92	2.83
		12.7	8.33	6.66	5.76	5.19	4.79	4.49	4.26	4.08	3.93
<b>D e g r e e s</b>	40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76
		4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
		7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
		12.6	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02	3.87

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# Table A.4

## Fisher's $f$ Distributions

(Continued From Previous Page)

First, second, third, and fourth rows correspond respectively to the upper tail probabilities 0.10, 0.05, 0.01, 0.001.

Columns are labeled by tail probabilities of the type corresponding to the shaded region in the figure on the right.

Table entries are  $f$  values.

		<b>N u m e r a t o r   D e g r e e s   o f   F r e e d o m</b>									
		1	2	3	4	5	6	7	8	9	10
<b>D e n o m i n a t o r</b>	42	2.83	2.43	2.22	2.08	1.99	1.92	1.86	1.82	1.78	1.75
		4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.06
		7.28	5.15	4.29	3.80	3.49	3.27	3.10	2.97	2.86	2.78
		12.5	8.18	6.53	5.64	5.07	4.68	4.38	4.16	3.97	3.83
<b>D e g r e e s</b>	44	2.82	2.43	2.21	2.08	1.98	1.91	1.86	1.81	1.78	1.75
		4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05
		7.25	5.12	4.26	3.78	3.47	3.24	3.08	2.95	2.84	2.75
		12.4	8.12	6.48	5.59	5.02	4.63	4.34	4.11	3.93	3.78
<b>D e g r e e s</b>	46	2.82	2.42	2.21	2.07	1.98	1.91	1.85	1.81	1.77	1.74
		4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.15	2.09	2.04
		7.22	5.10	4.24	3.76	3.44	3.22	3.06	2.93	2.82	2.73
		12.4	8.06	6.42	5.54	4.98	4.59	4.30	4.07	3.89	3.74
<b>D e g r e e s</b>	48	2.81	2.42	2.20	2.07	1.97	1.90	1.85	1.80	1.77	1.73
		4.04	3.19	2.80	2.57	2.41	2.29	2.21	2.14	2.08	2.03
		7.19	5.08	4.22	3.74	3.43	3.20	3.04	2.91	2.80	2.71
		12.3	8.00	6.38	5.50	4.94	4.55	4.26	4.03	3.85	3.70
<b>D e g r e e s</b>	50	2.81	2.41	2.20	2.06	1.97	1.90	1.84	1.80	1.76	1.73
		4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03
		7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70
		12.2	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82	3.67
<b>D e g r e e s</b>	60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71
		4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
		7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
		12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69	3.54
<b>D e g r e e s</b>	70	2.78	2.38	2.16	2.03	1.93	1.86	1.80	1.76	1.72	1.69
		3.93	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97
		7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59
		11.8	7.64	6.06	5.20	4.66	4.28	3.99	3.77	3.60	3.45

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## Table A.4

### Fisher's $f$ Distributions

(Continued From Previous Page)

First, second, third, and fourth rows correspond respectively to the upper tail probabilities 0.10, 0.05, 0.01, 0.001.

Columns are labeled by tail probabilities of the type corresponding to the shaded region in the figure on the right.

Table entries are  $f$  values.

		<b>N u m e r a t o r   D e g r e e s   o f   F r e e d o m</b>										
		1	2	3	4	5	6	7	8	9	10	
<b>D</b>	<b>e</b>	80	2.77	2.37	2.15	2.02	1.92	1.85	1.79	1.75	1.71	1.68
			3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95
			6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55
			11.7	7.54	5.97	5.12	4.58	4.20	3.92	3.70	3.53	3.39
<b>n</b>	<b>a</b>	90	2.76	2.36	2.15	2.01	1.91	1.84	1.78	1.74	1.70	1.67
			3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94
			6.93	4.85	4.01	3.53	3.23	3.01	2.84	2.72	2.61	2.52
			11.6	7.47	5.91	5.06	4.53	4.15	3.87	3.65	3.48	3.34
<b>r</b>	<b>e</b>	100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69	1.66
			3.94	3.09	2.70	2.45	2.31	2.19	2.10	2.03	1.97	1.93
			6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50
			11.5	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44	3.30
<b>s</b>	<b>o</b>	1000	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64	1.61
			3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84
			6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34
			10.9	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13	2.99
<b>o</b>		<b>f</b>		<b>F</b>		<b>r</b>		<b>e</b>		<b>e</b>		
<b>d</b>		<b>o</b>		<b>m</b>		<b>o</b>		<b>f</b>		<b>m</b>		

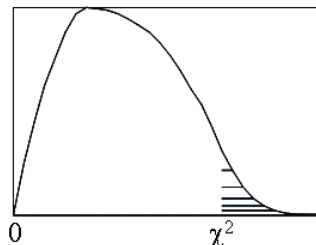
## Table A.5

### Chi-Square ( $\chi^2$ ) Distributions

Rows are labeled by  $df$  (degrees of freedom).

Columns are labeled by tail probabilities of the type corresponding to the shaded region in the figure on the right.

Table entries are  $\chi^2$  values.



$df$	0.10	0.05	0.025	0.01	0.005	0.001
1	2.706	3.841	5.024	6.635	7.879	10.828
2	4.605	5.991	7.378	9.210	10.597	13.816
3	6.251	7.815	9.348	11.345	12.838	16.266
4	7.779	9.488	11.143	13.277	14.860	18.467
5	9.236	11.070	12.833	15.086	16.750	20.515
6	10.645	12.592	14.449	16.812	18.548	22.458
7	12.017	14.067	16.013	18.475	20.278	24.322
8	13.362	15.507	17.535	20.090	21.955	26.125
9	14.684	16.919	19.023	21.666	23.589	27.877
10	15.987	18.307	20.483	23.209	25.188	29.588
11	17.275	19.675	21.920	24.725	26.757	31.264
12	18.549	21.026	23.337	26.217	28.300	32.909
13	19.812	22.362	24.736	27.688	29.819	34.528
14	21.064	23.685	26.119	29.141	31.319	36.123
15	22.307	24.996	27.488	30.578	32.801	37.697
16	23.542	26.269	28.845	32.000	34.267	39.252
17	24.769	27.587	30.191	33.409	35.718	40.790
18	25.989	28.869	31.526	34.805	37.156	42.312
19	27.204	30.144	32.852	36.191	38.582	43.820
20	28.412	31.410	34.170	37.566	39.997	45.315
21	29.615	32.671	35.479	38.932	41.401	46.797
22	30.813	33.924	36.781	40.289	42.796	48.268
23	32.007	35.172	38.076	41.638	44.181	49.728
24	33.196	36.415	39.364	42.980	45.559	51.179
25	34.382	37.652	40.646	44.314	46.928	52.620
26	35.563	38.885	41.923	45.642	48.290	54.052
27	36.741	40.113	43.195	46.963	49.645	55.476
28	37.916	41.337	44.461	48.278	50.993	56.829
29	39.087	42.557	45.722	49.588	52.336	58.302
30	40.256	43.773	46.979	50.892	53.672	59.703
40	51.805	55.758	59.342	63.691	66.766	73.402
50	63.167	67.505	71.420	76.154	79.490	86.661
60	74.397	79.082	83.298	88.379	91.952	99.607
80	96.578	101.879	106.629	112.329	116.321	124.839
100	118.498	124.342	129.561	135.807	140.169	149.449

(Continued Next Page →)

## Table A.5

### Chi-Square ( $\chi^2$ ) Distributions

*(Continued From Previous Page)*

Rows are labeled by  $df$  (degrees of freedom).

Columns are labeled by tail probabilities of the type corresponding to the shaded region in the figure on the right.

Table entries are  $\chi^2$  values.

$df$	0.995	0.99	0.975	0.95	0.90
1	0.0001	0.0002	0.001	0.004	0.016
2	0.010	0.020	0.051	0.103	0.211
3	0.072	0.115	0.216	0.352	0.584
4	0.207	0.297	0.484	0.711	1.064
5	0.412	0.554	0.831	1.145	1.610
6	0.676	0.872	1.237	1.635	2.204
7	0.989	1.239	1.690	2.167	2.833
8	1.344	1.646	2.180	2.733	3.490
9	1.735	2.088	2.700	3.325	4.168
10	2.156	2.558	3.247	3.940	4.865
11	2.603	3.053	3.816	4.575	5.578
12	3.074	3.571	4.404	5.226	6.304
13	3.565	4.107	5.009	5.892	7.042
14	4.075	4.660	5.629	6.571	7.790
15	4.601	5.229	6.262	7.261	8.547
16	5.142	5.812	6.908	7.962	9.312
17	5.679	6.408	7.564	8.672	10.085
18	6.265	7.015	8.231	9.390	10.865
19	6.844	7.633	8.907	10.117	11.651
20	7.434	8.260	9.591	10.851	12.443
21	8.034	8.897	10.283	11.591	13.240
22	8.643	9.542	10.982	12.338	14.041
23	9.260	10.196	11.689	13.091	14.848
24	9.886	10.856	12.401	13.848	15.659
25	10.520	11.524	13.120	14.611	16.473
26	11.160	12.198	13.844	15.379	17.292
27	11.808	12.879	14.573	16.151	18.144
28	12.461	13.565	15.308	16.928	18.939
29	13.121	14.256	16.047	17.708	19.768
30	13.787	14.953	16.791	18.493	20.599
40	20.707	22.164	24.433	26.509	29.501
50	27.991	29.707	32.357	34.764	37.689
60	35.534	37.485	40.482	43.188	46.459
80	51.172	53.540	57.153	60.391	64.278
100	67.328	70.065	74.222	77.929	82.358

# **Appendix B - Derivations and Formulas**

## Section B.1

### The Multiplication Principle

## Section B.2

### Calculation of the Two Sample $t$ Test Statistics

## Section B.3

### Calculation of Fisher's One-Way ANOVA $f$ Statistic

## Section B.1

# The Multiplication Principle

The *multiplication principle* gives us the number of results which are possible with an ordered sequence of actions. Suppose you are about to order a quick lunch at a cafeteria, and you must first choose one of 5 sandwiches (hamburger, hot dog, grilled cheese, hoagie, tuna fish) after which you must choose one of 3 drinks (soda, juice, milk). How many different combinations are there for lunch? If you match each one of the 5 sandwich choices with each one of the 3 drink choices, you find that there are  $(5)(3) = 15$  lunch combinations. (Go ahead and write down all the combinations to see that this is true!) The multiplication principle states that if there are  $w_1$  possible results for a first action and  $w_2$  possible results for a second action, then there are  $w_1w_2$  different results for both actions together performed in sequence.

The multiplication principle can be extended to any number of actions performed in sequence. Suppose that for your quick lunch in the cafeteria, you must first choose one of the 5 sandwiches (hamburger, hot dog, grilled cheese, hoagie, tuna fish), you must then choose one of the 3 drinks (soda, juice, milk), and finally you must choose one of 4 desserts (pie, ice cream, cheesecake, jello). How many different combinations are there for lunch? If you match each one of the 5 sandwich choices with each one of the 3 drink choices, then match each of these pairs with each one of the 4 dessert choices, you find that there are  $(5)(3)(4) = 60$  lunch combinations. (It may take a few minutes, but you can write down all the combinations to see that this is true!) In general, the multiplication principle states that if there are  $w_1$  possible results for a first action,  $w_2$  possible results for a second action, ...,  $w_k$  possible results for a  $k$ th action, then there are  $w_1w_2\dots w_k$  different results for all  $k$  actions performed together in sequence.

Now, let us consider a cube with four of the sides each painted red and the other two sides each painted blue. In order to use the multiplication principle to obtain a list of all the possible samples of size  $n = 3$ , we first observe that on each roll there are 4 red sides which can be facing up and 2 blue sides which can be facing up. To enumerate the number of possible samples, we ask the following eight questions: How many samples will result in

- (i) a red side facing up for all three rolls, which we designate as RRR?
- (ii) a blue side facing up on the first roll, and a red side facing up for the second and third rolls, which we designate as BRR?
- (iii) a red side facing up for the first and third rolls, and a blue side facing up on the second roll, which we designate as RBR?
- (iv) a red side facing up for the first and second rolls, and a blue side facing up on the third roll, which we designate as RRB?
- (v) a red side facing up on the first roll, and a blue side facing up for the second and third rolls, which we designate as RBB?
- (vi) a blue side facing up for the first and third rolls, and a red side facing up on the second roll, which we designate as BRB?

- (vii) a blue side facing up for the first and second rolls, and a red side facing up on the third roll, which we designate as BBR?
- (viii) a blue side facing up for all three rolls, which we designate as BBB?

We can use the multiplication principle to answer each question by realizing that on each roll, there are four ways a red side can be facing up and two ways a blue side can be facing up.

Since there are 4 ways to obtain a red side on the first roll, 4 ways to obtain a red side on the second roll, and 4 ways to obtain a red side on the third roll, then the multiplication principle tells us that there must be  $(4)(4)(4) = 64$  ways to obtain the result designated as RRR; this answers question (i). This result is recorded in the first row of Table 18-1, where we also find that the sample proportion of red is  $3/3 = 1.000$ .

Since there are 2 ways to obtain a blue side on the first roll, 4 ways to obtain a red side on the second roll, and 4 ways to obtain a red side on the third roll, then the multiplication principle tells us that there must be  $(2)(4)(4) = 32$  ways to obtain the result designated as BRR; this answers question (ii). This result is recorded in the second row of Table 18-1, where we also find that the sample proportion of red is  $2/3 = 0.667$ .

Since there are 4 ways to obtain a red side on the first roll, 2 ways to obtain a blue side on the second roll, and 4 ways to obtain a red side on the third roll, then the multiplication principle tells us that there must be  $(4)(2)(4) = 32$  ways to obtain the result designated as RBR; this answers question (iii). This result is recorded in the third row of Table 18-1, where we also find that the sample proportion of red is  $2/3 = 0.667$ .

Since there are 4 ways to obtain a red side on the first roll, 4 ways to obtain a red side on the second roll, and 2 ways to obtain a blue side on the third roll, then the multiplication principle tells us that there must be  $(4)(4)(2) = 32$  ways to obtain the result designated as RRB; this answers question (iv). This result is recorded in the fourth row of Table 18-1, where we also find that the sample proportion of red is  $2/3 = 0.667$ .

Since there are 4 ways to obtain a red side on the first roll, 2 ways to obtain a blue side on the second roll, and 2 ways to obtain a blue side on the third roll, then the multiplication principle tells us that there must be  $(4)(2)(2) = 16$  ways to obtain the result designated as RBB; this answers question (v). This result is recorded in the fifth row of Table 18-1, where we also find that the sample proportion of red is  $1/3 = 0.333$ .

Since there are 2 ways to obtain a blue side on the first roll, 4 ways to obtain a red side on the second roll, and 2 ways to obtain a blue side on the third roll, then the multiplication principle tells us that there must be  $(2)(4)(2) = 16$  ways to obtain the result designated as BRB; this answers question (vi). This result is recorded in the sixth row of Table 18-1, where we also find that the sample proportion of red is  $1/3 = 0.333$ .

Since there are 2 ways to obtain a blue side on the first roll, 2 ways to obtain a blue side on the second roll, and 4 ways to obtain a red side on the third roll, then the multiplication principle tells us that there must be  $(2)(2)(4) = 16$  ways to obtain the result designated as BBR; this answers question (vii). This result is recorded in the seventh row of Table 18-1, where we also find that the sample proportion of red is  $1/3 = 0.333$ .

Since there are 2 ways to obtain a blue side on the first roll, 2 ways to obtain a blue side on the second roll, and 2 ways to obtain a blue side on the third roll, then the multiplication principle tells us that there must be  $(2)(2)(2) = 8$  ways to obtain the result designated as BBB; this answers question (viii). This result is recorded in the eighth row of Table 18-1, where we also find that the sample proportion of red is  $0/3 = 0.000$ .

## Section B.2

### Calculation of the Two Sample $t$ Test Statistics

Notation & Formulas	Example						
<p>Data consisting of two independent samples of sizes <math>n_1</math> and <math>n_2</math> can be represented as follows:</p> <p>Sample #1                      Sample #2</p> <p><math>x_{11}, x_{12}, \dots, x_{1n_1}</math>      <math>x_{21}, x_{22}, \dots, x_{2n_2}</math></p> <p>(The first subscript indicates which sample the observation is from, and the second subscript distinguishes between different observations in the same sample.)</p> <p><math>\sum x_1</math> and <math>\sum x_2</math> respectively represent the sum of the observations in sample #1 and the sum of the observations in sample #2, from which the sample means can be obtained as follows:</p> $\bar{x}_1 = \frac{\sum x_1}{n_1} \text{ and } \bar{x}_2 = \frac{\sum x_2}{n_2} .$ <p><math>\sum(x_1 - \bar{x}_1)^2</math> and <math>\sum(x_2 - \bar{x}_2)^2</math> respectively represent the sum of the squared deviations from the mean in sample #1 and the sum of the squared deviations from the mean in sample #2, from which the sample variances can be obtained as follows:</p> $s_1^2 = \frac{\sum(x_1 - \bar{x}_1)^2}{n_1 - 1} \text{ and } s_2^2 = \frac{\sum(x_2 - \bar{x}_2)^2}{n_2 - 1} .$ <p>Note that if the sample variances <math>s_1^2</math> and <math>s_2^2</math> are available, then the sums of squared deviations can be obtained from</p> $\sum(x_1 - \bar{x}_1)^2 = (n_1 - 1)s_1^2 \text{ and}$ $\sum(x_2 - \bar{x}_2)^2 = (n_2 - 1)s_2^2 .$	<p>The hours of lifetime is recorded for each of four bulbs from the brand named Sunn and three bulbs from the brand named Brighto, resulting in the following data consisting of two independent samples:</p> <table style="width: 100%; border: none;"> <tr> <td style="text-align: center; border-bottom: 1px solid black;"><u>Sunn</u></td> <td style="text-align: center; border-bottom: 1px solid black;"><u>Brighto</u></td> </tr> <tr> <td style="text-align: center;">575 611 572 602</td> <td style="text-align: center;">568 569 528</td> </tr> <tr> <td style="text-align: center;">(<math>n_S = 4</math>)</td> <td style="text-align: center;">(<math>n_B = 3</math>)</td> </tr> </table> <p><math>\sum x_S = 575 + 611 + 572 + 602 = 2360</math></p> <p><math>\sum x_B = 568 + 569 + 528 = 1665</math></p> <p><math>\bar{x}_S = \frac{2360}{4} = 590</math> and <math>\bar{x}_B = \frac{1665}{3} = 555</math></p> <p><math>\sum(x_S - \bar{x}_S)^2 = (575 - 590)^2 + (611 - 590)^2 + (572 - 590)^2 + (602 - 590)^2 = 1134</math></p> <p><math>\sum(x_B - \bar{x}_B)^2 = (568 - 555)^2 + (569 - 555)^2 + (528 - 555)^2 = 1094</math></p> <p><math>s_S^2 = \frac{1134}{4-1} = 378</math></p> <p><math>s_B^2 = \frac{1094}{3-1} = 547</math></p>	<u>Sunn</u>	<u>Brighto</u>	575 611 572 602	568 569 528	( $n_S = 4$ )	( $n_B = 3$ )
<u>Sunn</u>	<u>Brighto</u>						
575 611 572 602	568 569 528						
( $n_S = 4$ )	( $n_B = 3$ )						
<i>(continued next page)</i>							

## Section B.2 (continued)

The pooled two sample  $t$  statistic requires the calculation of the pooled sample variance

$$s_p^2 = \frac{\sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2} .$$

The pooled two sample  $t$  statistic, with  $df = n_1 + n_2 - 2$ , is

$$t_{n_1 + n_2 - 2} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} .$$

The separate two sample  $t$  statistic requires the calculation of the degrees of freedom

$$v = \frac{(n_1 - 1)(n_2 - 1) \left( \frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)^2}{(n_2 - 1) \left( \frac{s_1^2}{n_1} \right)^2 + (n_1 - 1) \left( \frac{s_2^2}{n_2} \right)^2} .$$

The separate two sample  $t$  statistic is

$$t_v = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} .$$

$$s_p^2 = \frac{\sum (x_S - \bar{x}_S)^2 + \sum (x_B - \bar{x}_B)^2}{n_S + n_B - 2} =$$

$$\frac{1134 + 1094}{4 + 3 - 2} = \frac{2228}{5} = 445.6$$

$$t_5 = \frac{\bar{x}_S - \bar{x}_B}{\sqrt{s_p^2 \left( \frac{1}{n_S} + \frac{1}{n_B} \right)}} = \frac{590 - 555}{\sqrt{445.6 \left( \frac{1}{4} + \frac{1}{3} \right)}} =$$

2.171

$$v = \frac{(n_S - 1)(n_B - 1) \left( \frac{s_S^2}{n_S} + \frac{s_B^2}{n_B} \right)^2}{(n_B - 1) \left( \frac{s_S^2}{n_S} \right)^2 + (n_S - 1) \left( \frac{s_B^2}{n_B} \right)^2} = 3.9 \approx$$

4

$$t_4 = \frac{\bar{x}_S - \bar{x}_B}{\sqrt{\frac{s_S^2}{n_S} + \frac{s_B^2}{n_B}}} = \frac{590 - 555}{\sqrt{\frac{378}{4} + \frac{547}{3}}} = +2.104$$

## Section B.3

### Calculation of Fisher's One-Way ANOVA $f$ Statistic

Notation & Formulas	Example
<p>Data consisting of <math>k</math> independent samples of sizes <math>n_1, n_2, \dots, n_k</math> can be represented as follows:</p> <p>Sample #1                      ...                      Sample #<math>k</math></p> <p><math>x_{11}, x_{12}, \dots, x_{1n_1}</math>                      ...                      <math>x_{k1}, x_{k2}, \dots, x_{kn_k}</math></p> <p>(The first subscript indicates which sample the observation is from, and the second subscript distinguishes between different observations in the same sample.)</p> <p><math>\sum x_i</math> represents the sum of the observations in sample #<math>i</math>, from which the sample means can be obtained as follows:</p> <p><math>\bar{x}_i = \frac{\sum x_i}{n_i}</math> is the sample mean for sample #<math>i</math>.</p> <p><math>n_{\bullet} = n_1 + n_2 + \dots + n_k =</math> total sample size</p> <p><math>\bar{x}_{\bullet} = \frac{\sum x_1 + \sum x_2 + \dots + \sum x_k}{n_{\bullet}} =</math> grand mean</p> <p><math>SSB = \sum n_i (\bar{x}_i - \bar{x}_{\bullet})^2</math> and <math>MSB = \frac{SSB}{k-1}</math></p> <p><math>\sum (x_i - \bar{x}_i)^2</math> represents the sum of the squared deviations from the mean in sample #<math>i</math>, from which <math>SSE</math> can be obtained:</p> <p><math>SSE = \sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2 + \dots + \sum (x_k - \bar{x}_k)^2</math></p> <p>and <math>MSB = \frac{SSE}{n_{\bullet} - k}</math>.</p> <p>Note that if the sample variances <math>s_1^2, s_2^2, \dots, s_k^2</math> are available, then the sums of squared deviations for sample #<math>i</math> can be obtained from <math>\sum (x_i - \bar{x}_i)^2 = (n_i - 1)s_i^2</math>.</p> <p><math>f_{k-1, n_{\bullet} - k} = \frac{MSB}{MSE}</math></p>	<p>Breaking strength is recorded for each of three random samples of rope, one for rope brand Deluxe, one for rope brand Econ, and one for rope brand Nogood, resulting in the following data consisting of three independent samples:</p> <p><u>Deluxe</u> 165 162 159 162                      (<math>n_D = 4</math>)</p> <p><u>Econ</u>    156 163 158                              (<math>n_E = 3</math>)</p> <p><u>Nogood</u> 151 154 160                              (<math>n_N = 3</math>)</p> <p><math>\sum x_D = 165 + 162 + 159 + 162 = 648</math></p> <p><math>\sum x_E = 156 + 163 + 158 = 477</math></p> <p><math>\sum x_N = 151 + 154 + 160 = 465</math></p> <p><math>\bar{x}_D = \frac{648}{4} = 162</math> ,                      <math>\bar{x}_E = \frac{477}{3} = 159</math> ,</p> <p><math>\bar{x}_N = \frac{465}{3} = 155</math></p> <p><math>n_{\bullet} = n_D + n_E + n_N = 4 + 3 + 3 = 10</math></p> <p><math>\bar{x}_{\bullet} = \frac{648 + 477 + 465}{10} = \frac{1590}{10} = 159</math></p> <p><math>SSB = (4)(162 - 159)^2 + (3)(159 - 159)^2 + (3)(155 - 159)^2 = 84</math></p> <p><math>MSB = \frac{84}{3-1} = 42</math></p> <p><math>\sum (x_D - \bar{x}_D)^2 = (165 - 162)^2 + (162 - 162)^2 + (159 - 162)^2 + (162 - 162)^2 = 18</math></p> <p><math>\sum (x_E - \bar{x}_E)^2 = (156 - 159)^2 + (163 - 159)^2 + (158 - 159)^2 = 26</math></p> <p><math>\sum (x_N - \bar{x}_N)^2 = (151 - 155)^2 + (154 - 155)^2 + (160 - 155)^2 = 42</math></p> <p><math>SSE = 18 + 26 + 42 = 86</math></p> <p><math>MSB = \frac{86}{10-3} = 12.2857</math></p> <p><math>f_{2, 7} = \frac{42}{12.2857} = 3.42</math></p>