

Unit 14

Sampling Distributions

Objectives:

- To distinguish between a statistic and a parameter
- To understand how the sampling distribution for a statistic, such as the sample mean, is constructed from the parent population
- To see how center, dispersion, and shape compare for the distribution of a parent population and the sampling distribution for the sample mean with simple random sampling

We have previously defined inferential statistics to be the use of information gathered from a data set to draw a conclusion about a set of items larger than those on which the data was taken. The larger set of items about which we wish to draw one or more conclusions is what we have called a population, and the finite set of items selected from the population for the purpose of providing the data on which conclusions about the population are based, is what we have called a sample.

The conclusions that we draw often focus on some numerical characteristic of the population such as a percentage or a mean, and we shall need to distinguish between numerical summaries corresponding to a sample and numerical summaries corresponding to a population. For example, suppose that a sample of 500 households in a city is to be selected to be surveyed for the purpose of drawing a conclusion about the mean yearly income for the population of all households in the entire the city. We need to make a distinction between the mean yearly income for the sample of 500 households surveyed and the mean yearly income for the population of all households in the entire the city. The mean yearly income for the 500 households surveyed is a value that we can obtain by averaging the 500 incomes observed. The mean yearly income for the population of all households in the entire the city is a value that we cannot obtain, since we do not have access to every household in the entire the city; after all, if it were possible and practical to observe the income for every household in the entire the city, we would not need to select a sample of households for the purpose of drawing a conclusion about the mean yearly income for all households!

In statistical terminology, a *parameter* refers to a numerical summary corresponding to a population, and a *statistic* refers to a numerical summary corresponding to a sample. In general, a parameter is a numerical summary that we do not know, because we are unable to observe every item in the population. We draw some conclusion about the value of a population parameter from the value of a sample statistic. In the illustration where 500 households in a city are sampled for the purpose of drawing a conclusion about the mean yearly income for the population of all households in the entire the city, the mean yearly income for the 500 households sampled is a statistic, and the mean yearly income for all households in the city is a parameter.

Self-Test Problem 14-1. Forty Econo brand light bulbs are selected in order to make a conclusion about the mean lifetime for all Econo brand light bulbs.

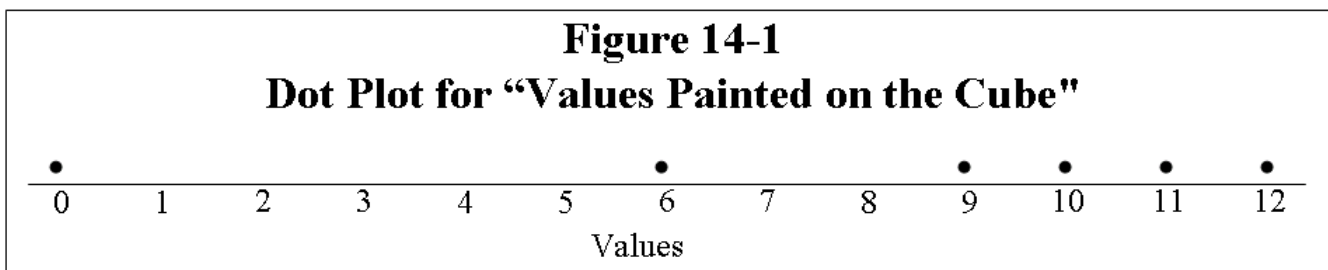
- (a) Identify the sample, population of interest, parameter of interest, and statistic used to estimate the parameter.
- (b) Are the population of interest and the accessible population the same? Why or why not?
- (c) Suppose the sample is selected by arbitrarily obtaining one bulb from those produced each day for forty days in a row. Is it reasonable to treat this as a simple random sample? Why or why not?

Notice that the value of a parameter is something that we do not and can not know for sure; however, we would like to estimate and/or draw conclusions about the value of the parameter, which is the purpose of inferential statistics. Notice also that the value of a statistic is something that we can obtain by just using our sample to do the appropriate calculation. The value of a parameter (though unknown) is a fixed value, whereas the value of a statistic will vary depending on the sample that is selected. In the illustration where 500 households in a city are sampled for the purpose of drawing a conclusion about the mean yearly income for the population of all households in the entire the city, the mean yearly income for all households in the city (a

parameter) is a fixed value; however, the value of the mean yearly income for the 500 households sampled (a statistic) will depend on which sample of 500 is selected.

In order to study the principles behind inferential statistics, we must study how the value of a statistic behaves when a sample is selected from a population. There are many numerical summaries that we could consider, but right now, we shall choose to focus on the mean. There are also many different sampling methods we could consider, but right now, we shall choose to focus on simple random sampling. For the sake of simplicity, the populations in our initial illustrations will be very small. While we know that in practice populations are very large, and potentially infinite, we shall be using some very small populations in the immediate future in order to illustrate some important concepts without having to do a lot of tedious computation.

As a simple illustration, let us suppose that a value is painted on each of the six sides of a small cube, but not the same values that you would see on an ordinary die. The values painted on the six sides of the cube are 0, 6, 9, 10, 11, and 12. Now, imagine that you have a friend who does not know what these values are and whose job it is to estimate what the mean of the values on the cube is. Figure 14-1 displays a dot plot of these values; notice that the distribution of "values painted on the cube" is negatively skewed.



If your friend knew what the six values were, then this friend could easily figure out that the mean is $(0+6+9+10+11+12)/6 = 48/6 = 8$. The reason why we are imagining that your friend does not know what the values are is to study how accurately your friend would be able to estimate the value of the mean using simple random sampling. This will hopefully provide you with some insight into the use of inferential statistics to draw conclusions about the value of a parameter corresponding to a population, based on the value of a statistic from a simple random sample. In our illustration, the population mean of the values on the cube is the parameter of interest, and the mean from a simple random sample of rolls is the statistic used to estimate the population mean.

Let us now imagine that your friend is permitted to roll the cube twice and record the value facing upward each time, without being able to see any of the other values. Essentially, your friend is selecting a simple random sample of size $n = 2$; your friend is going to use the sample mean to estimate the population mean. Before proceeding any further, we must define some notation to distinguish between the two different means that we are talking about here.

Back when we were exclusively focusing on descriptive statistics, we used the symbol \bar{x} to refer to the mean. From now on, we shall use the symbol \bar{x} only to refer to a sample mean. To refer to a population mean, we shall use the Greek letter μ (mu). In other words, \bar{x} is a statistic and μ is a parameter. In our present illustration with the cube, $\mu = 8$ is the mean of the population consisting of the values painted on the six sides of a small cube, and \bar{x} is the mean of the sample from $n = 2$ rolls of the cube. As we have stated earlier, a parameter such as μ is always a fixed, single value that we are trying to estimate because we do not know its value, and a statistic such as \bar{x} is a random value which depends on the sample we select.

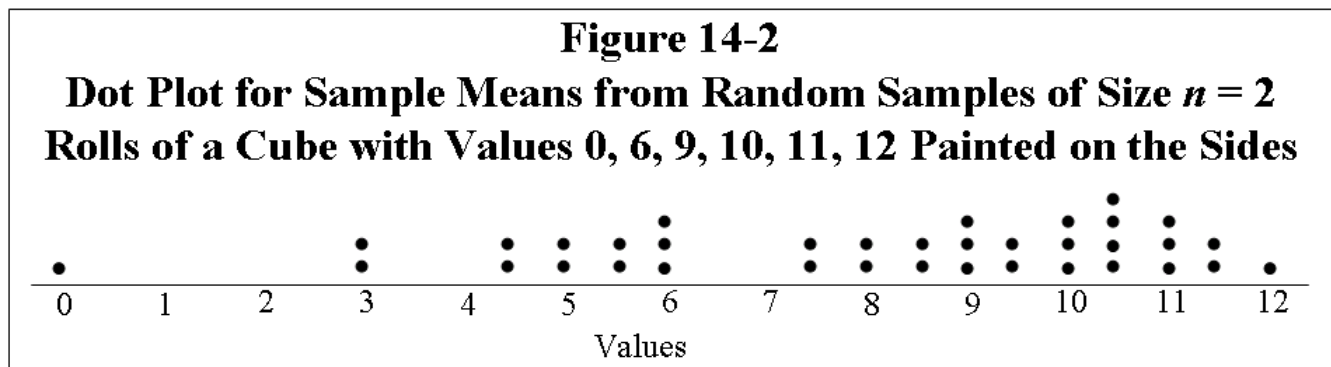
Let us now return to the illustration where your friend does not know that $\mu = 8$ but is going to estimate μ with the value of \bar{x} from a simple random sample of $n = 2$ rolls of the cube. In order to evaluate your friend's chances of obtaining an estimate of the population mean which is reasonably close to the correct value of 8, we shall first make a list of all the samples of size $n = 2$ which are possible.

We constructed the list of possible samples of size $n = 2$ in Table 14-1 by proceeding in an orderly fashion. The first two values listed in Table 14-1 are the two smallest values we could possibly observe with two rolls: two 0s. The second sample listed in Table 14-1 consists of a 0, which is the smallest value we could

observe in a single roll, and a 6, which is the second smallest value we could observe in a single roll. The third sample consists of a 0, which is the smallest value we could observe in a single roll, and a 9, which is the third smallest value we could observe in a single roll. By continuing in this fashion all six samples of size $n = 2$ where a 0 is observed first are listed; these are the first six samples listed in Table 14-1. Next, a 6 is paired with each of 0, 6, 9, 10, 11, and 12, resulting in six more samples of size $n = 2$. By continuing in this manner, all 36 samples of size of size $n = 2$ are found.

Table 14-1
Samples of Size $n = 2$ Rolls of a Cube with Sides 0, 6, 9, 10, 11, 12

<u>Samples of Size 2</u>	<u>Sample Mean</u>	<u>Samples of Size 2</u>	<u>Sample Mean</u>
0, 0	$(0 + 0) / 2 = 0$	10, 0	$(10 + 0) / 2 = 5$
0, 6	$(0 + 6) / 2 = 3$	10, 6	$(10 + 6) / 2 = 8$
0, 9	$(0 + 9) / 2 = 4.5$	10, 9	$(10 + 9) / 2 = 9.5$
0, 10	$(0 + 10) / 2 = 5$	10, 10	$(10 + 10) / 2 = 10$
0, 11	$(0 + 11) / 2 = 5.5$	10, 11	$(10 + 11) / 2 = 10.5$
0, 12	$(0 + 12) / 2 = 6$	10, 12	$(10 + 12) / 2 = 11$
6, 0	$(6 + 0) / 2 = 3$	11, 0	$(11 + 0) / 2 = 5.5$
6, 6	$(6 + 6) / 2 = 6$	11, 6	$(11 + 6) / 2 = 8.5$
6, 9	$(6 + 9) / 2 = 7.5$	11, 9	$(11 + 9) / 2 = 10$
6, 10	$(6 + 10) / 2 = 8$	11, 10	$(11 + 10) / 2 = 10.5$
6, 11	$(6 + 11) / 2 = 8.5$	11, 11	$(11 + 11) / 2 = 11$
6, 12	$(6 + 12) / 2 = 9$	11, 12	$(11 + 12) / 2 = 11.5$
9, 0	$(9 + 0) / 2 = 4.5$	12, 0	$(12 + 0) / 2 = 6$
9, 6	$(9 + 6) / 2 = 7.5$	12, 6	$(12 + 6) / 2 = 9$
9, 9	$(9 + 9) / 2 = 9$	12, 9	$(12 + 9) / 2 = 10.5$
9, 10	$(9 + 10) / 2 = 9.5$	12, 10	$(12 + 10) / 2 = 11$
9, 11	$(9 + 11) / 2 = 10$	12, 11	$(12 + 11) / 2 = 11.5$
9, 12	$(9 + 21) / 2 = 10.5$	12, 12	$(12 + 12) / 2 = 12$



With simple random sampling, each of the 36 samples listed in Table 14-1 has an equal chance (1 out of 36) of being the one which your friend will obtain with $n = 2$ rolls of the cube and consequently the one from which your friend's estimate of μ will come. In order to evaluate your friend's chances of obtaining an estimate of the population mean which is reasonably close to the correct value of $\mu = 8$, we consider the distribution of

possible sample means, that is, the distribution of possible values for \bar{x} . Figure 14-2 displays a dot plot of the equally likely possible values for \bar{x} , constructed from the values of \bar{x} listed in Table 14-1.

The dot plots of Figures 14-1 and 14-2 are based on a different number of values; in order to compare these two distributions more easily, we can first construct the frequency distributions displayed in Tables 14-2a and 14-2b. The frequency distribution of Table 14-2a displays the relative frequencies for a single roll of the cube. The frequency distribution of Table 14-2b represents what we call the *sampling distribution of \bar{x}* based on simple random sampling; in other words, the sampling distribution of \bar{x} is a frequency distribution displaying the possible values of \bar{x} together with the corresponding relative frequency of times each value occurs with simple random sampling. Figure 14-3a is a histogram of the relative frequencies for a single roll of the cube, as listed in Table 14-2a, and Figure 14-3b is a histogram displaying the relative frequencies in the sampling distribution of \bar{x} with $n = 2$ rolls, as listed in Table 14-2b.

As you compare Figures 14-3a and 14-3b, you should notice two things. First, you should realize that although the values from a single roll and the values of the sample mean resulting from $n = 2$ rolls both range from 0 and 12, there are a larger number of possible values for the sample mean than there are for a single roll; specifically, a single roll can only result in 0, 6, 9, 10, 11, or 12, whereas the sample mean from $n = 2$ rolls can result in 0, 3, 4.5, 5, 5.5, 6, 7.5, 8, 8.5, 9, 9.5, 10, 10.5, 11, 11.5, or 12. The second thing to notice is that each of the two extreme values of 0 and 12 are considerably less likely to occur with the sample mean from $n = 2$ rolls than with a single roll; in particular, with a single roll, the chance of obtaining a 0 is $1/6 = 16.7\%$ as is the chance of obtaining a 12, whereas with the sample mean from $n = 2$ rolls, the chance of obtaining a 0 is $1/36 = 2.8\%$ as is the chance of obtaining a 12. This suggests that with the sample mean from $n = 2$ rolls, it is more likely to observe values in between the two extremes than it is with a single roll.

We see then that your friend's chances of obtaining an estimate of the population mean which is reasonably close to the correct value of $\mu = 8$ are considerably better with the sample mean from $n = 2$ rolls than with a single roll. This fact by itself is not really a surprise. You could have undoubtedly deduced this without our going through the process of constructing the distribution for a single roll and the sampling distribution of \bar{x} with $n = 2$. What is surprising, though, is just how drastically different the sampling distribution of \bar{x} with $n = 2$ is from the distribution resulting from a single roll. These differences become even more pronounced as the number of rolls n (i.e., the sample size) is increased.

It would be an extremely time consuming task to list by hand all the samples resulting from $n = 3$ rolls, or from $n = 4$ rolls, etc. If one were to look at the sampling distribution of \bar{x} with each of these sample sizes, however, one would find that values near 0 and 12 become less likely, whereas values near $\mu = 8$ become more likely. With the aid of a computer, we did construct the sampling distribution of \bar{x} with $n = 6$, which is

Table 14-2a
Distribution of "Values Painted on a Six-Sided Cube"

<u>Value</u>	<u>Relative Frequency</u>
0	$1/6 = 16.7\%$
6	$1/6 = 16.7\%$
9	$1/6 = 16.7\%$
10	$1/6 = 16.7\%$
11	$1/6 = 16.7\%$
12	$1/6 = 16.7\%$

Table 14-2b
Sampling Distribution of \bar{x} with $n = 2$ Rolls

<u>Sample Mean</u>	<u>Relative Frequency</u>
0	$1/36 = 2.8\%$
3	$2/36 = 5.6\%$
4.5	$2/36 = 5.6\%$
5	$2/36 = 5.6\%$
5.5	$2/36 = 5.6\%$
6	$3/36 = 8.3\%$
7.5	$2/36 = 5.6\%$
8	$2/36 = 5.6\%$
8.5	$2/36 = 5.6\%$
9	$3/36 = 8.3\%$
9.5	$2/36 = 5.6\%$
10	$3/36 = 8.3\%$
10.5	$4/36 = 11.1\%$
11	$3/36 = 8.3\%$
11.5	$2/36 = 5.6\%$
12	$1/36 = 2.8\%$

displayed as a histogram in Figure 14-4. Notice that because of the large number of possible values for the sample mean from $n = 6$ rolls, we defined the categories "From 0 to 1," "Above 1 to 2," "Above 2 to 3," etc.

The histogram of Figure 14-4 visually displays just how unlikely it is that the sample mean from $n = 6$ rolls will be very far away from $\mu = 8$, even though $n = 6$ would be considered a relatively small sample size for most practical situations. Returning to Figures 14-3a and 14-3b, we see that the distribution resulting from a single roll is negatively skewed, and that the sampling distribution of \bar{x} with $n = 2$ is also negatively skewed. Surprisingly, though, the sampling distribution of \bar{x} with $n = 6$ looks bell-shaped and almost symmetric! As the sample size is increased, not only do the extreme values in the range become less likely in the sampling distribution of \bar{x} , but the sampling distributions become more symmetric and bell-shaped (i.e., such as in Figure 3-5b). We would find that the sampling distributions of \bar{x} for $n > 6$ are practically bell-shaped with values near $\mu = 8$ becoming more likely as n increases.

Let us now review and summarize what we have learned about sampling distributions, and at the same time, define some notation which will be useful later on. When we consider selecting a sample of size n , the population from which the sample is to be selected can be called the *parent population*.

In the illustration where the integers 0, 6, 9, 10, 11, and 12 are painted on the six sides of a cube, these six integers comprised our parent population.

The parameter of interest was the population mean μ , and since the six integers are equally likely to occur, we found that $\mu = 8$. The sample mean \bar{x} is the statistic used to estimate μ . For each sample size n , we have a population consisting of the possible values of \bar{x} . With simple random sampling, each sample of size n is equally likely to be selected, and it is this fact which determines the sampling distribution of \bar{x} .

When we consider the sampling distribution of \bar{x} , we are actually considering many different populations simultaneously: the parent population and each of the populations consisting of the possible values of \bar{x} corresponding to a particular n . Since we use μ to represent the mean corresponding to the parent population, we shall use $\mu_{\bar{x}}$ to represent the mean corresponding to the population consisting of the possible values of \bar{x} from a given random sample size n , that is, $\mu_{\bar{x}}$ represents the mean of the sampling distribution of \bar{x} with random samples of size n .

In the illustration where the integers 0, 6, 9, 10, 11, and 12 are painted on the six sides of a cube, the mean of the parent population was easily seen to be

$$\mu = (0 + 6 + 9 + 10 + 11 + 12) / 6 = 8.$$

Figure 14-3a

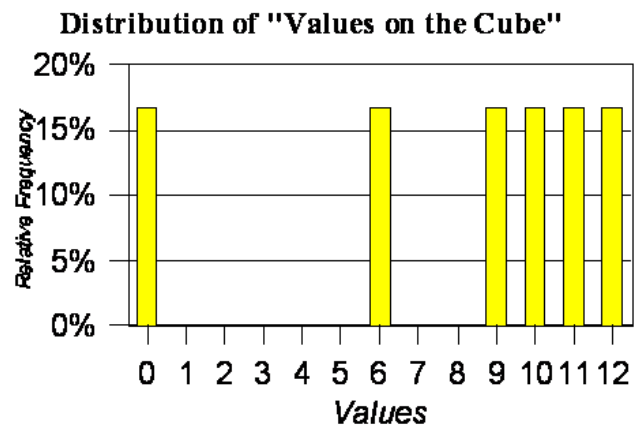
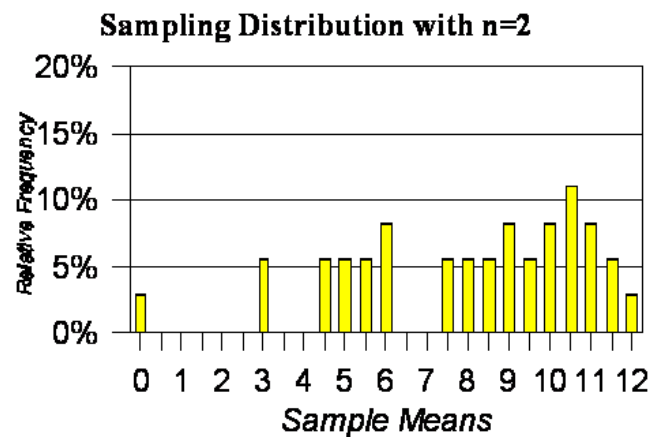


Figure 14-3b



Let us now find $\mu_{\bar{x}}$ for $n = 2$. All the possible values of \bar{x} with samples of size $n = 2$ are listed in Table 14-1, and we could simply add these 36 values and divide this sum by 36. However, it will be easier to use the raw frequencies which are available from Table 14-2b. From Table 14-2b, we find that one sample mean is equal to 0, two sample means are equal to 3, two sample means are equal to 4.5, etc. Using this information, we find that

$$\mu_{\bar{x}} = [(1)(0)+(2)(3)+(2)(4.5)+\dots+(1)(12)]/36 = 288/36 = 8 .$$

We see then that the population mean of all sample means with $n = 2$ is equal to the mean of the parent population. It should not be surprising that this will always be true for all sample sizes n . That is, $\mu_{\bar{x}} = \mu$ for any sample size n .

When we compared the parent population (consisting of the integers 0, 6, 9, 10, 11, and 12), the sampling distribution of \bar{x} for $n = 2$ rolls, and the sampling distribution of \bar{x} for $n = 6$ rolls, we found that as n increases the extreme values become less likely and the values near $\mu = 8 = \mu_{\bar{x}}$ become more likely. The fact that the extreme values become less likely as the sample size n increases is a reflection of the fact that there is less dispersion in the sampling distributions as the sample size n increases, even though the sampling distributions all have the same center; in addition, recall that we also noticed the sampling distributions become more bell-shaped as n increases.

If you think carefully, about what happens as the sample size n increases, the fact that the extreme values become less likely is not really surprising. If you roll the cube once, the chance of observing a 0 is equal to the chance of observing any one of the other five values. However, if you were to roll the cube six times, the only way you could observe a sample mean close to 0 would be to have practically every one of the six rolls be a 0, which is very unlikely. On the other hand, there are many different ways to observe a sample mean in the middle of the range between the extreme values of 0 and 12.

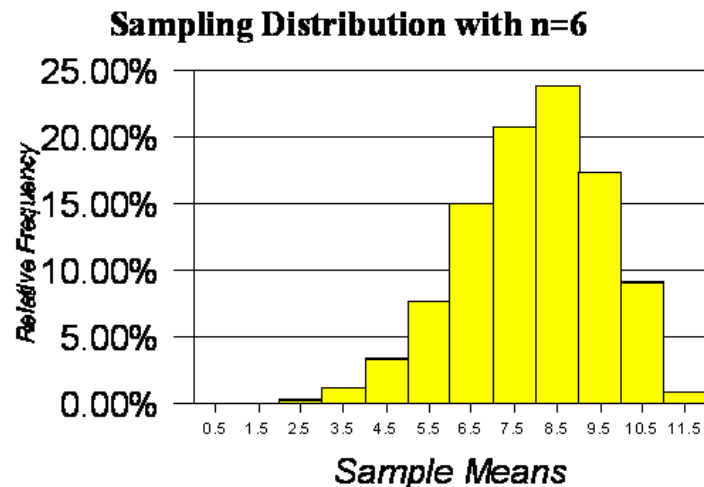
What we have observed in the illustration with the cube is a reflection of what is true in many practical situations. When simple random samples of size n are selected from a parent population with mean μ and a finite variance, then the following three statements will be true:

- (1) $\mu_{\bar{x}} = \mu$.
- (2) The variation in the sampling distribution of \bar{x} is less than in the parent population and decreases as n increases.
- (3) The sampling distribution of \bar{x} becomes more bell-shaped decreases as n increases.

We shall find these three facts to be very important in the future, and later on, we shall see how we can state these three facts in a more precise manner. (Notice that we have stated that the parent population must have a finite variance. There do exist some special types of parent populations which actually have an infinite variance; for these types of populations, the three statements concerning sampling distributions do not apply. However, we shall not be concerned with these types of populations in the immediate future.)

In our discussions of different distributions, the phrase *relative frequency* of course refers to the proportion or percentage corresponding to one or more values. For instance, in Table 14-2b we find that the relative frequency corresponding to a sample mean of 10.5 in two rolls is $4/36 = 11.1\%$, which we interpret as

Figure 14-4



follows. If the act of rolling the cube twice and computing the sample mean were done repeatedly, then a sample mean equal to 10.5 would be observed about 11.1% of the time in a large number of repetitions. In other words, 11.1% is the long term relative frequency of times a sample mean equal to 10.5 would be observed.

It certainly makes sense to define the long term relative frequency of a particular occurrence to be the *probability* of that occurrence, since it is natural and intuitive to think of probability as the long term proportion or percentage of times a particular event occurs. With this in mind, we shall from now on use the word *probability* and the phrase *relative frequency* interchangeably.

Type	Frequency	Salary(\$1000s)
Factory Worker	40	20
Salesperson	30	40
Managerial	20	60
Vice President	10	80

Self-Test Problem 14-2. Table 14-3 contains some information about the 100 employees of the Econo Corporation. Suppose these 100 employees represent the parent population from which a simple random sample is to be selected using Table A.1 (the random number table). The 40 factory workers are labeled 01, 02, ..., 40; the 30 salespeople are labeled 41, 42, ..., 70; the 20 managers are labeled 71, 72, ..., 90; the 10 vice presidents are labeled 91, 92, ..., 99, 00.

- Construct a histogram for the salaries in the parent population with salary on the horizontal axis and relative frequency on the vertical axis.
- Find the mean salary (μ) in the parent population.
- Describe the shape for the distribution of salaries in the parent population.
- Suppose we were to list all possible samples of $n = 4$ employee labels (but do not actually do this), then obtain the sample mean salary (\bar{x}) for each of these possible samples of size $n = 4$, and finally construct a histogram for these sample mean salaries. The histogram would display the sampling distribution of \bar{x} with simple random samples of size $n = 4$; if we were to calculate the value of the mean ($\mu_{\bar{x}}$) for this sampling distribution (but you do not need to actually do this!), what would we find the value to be?
- How would the dispersion in the sampling distribution of \bar{x} with simple random samples of size $n = 4$ compare with the dispersion in the distribution of salaries in the parent population?
- How would the shape of the sampling distribution of \bar{x} with simple random samples of size $n = 4$ compare with the shape of the distribution of salaries in the parent population?
- Use Table A.1 to select a simple random sample of $n = 4$ employee labels by reading the first two digits of each set of five digits starting in any row and finding the sample mean (\bar{x}) of the salaries of the selected employees; then, repeat this 9 more times with 9 other different rows to obtain a total of 10 simple random sample means. Construct a histogram for the 10 sample mean salaries, with salary on the horizontal axis and relative frequency on the vertical axis. Are the center of the distribution, the dispersion of the distribution, and the shape of the distribution consistent with the answers to parts (d), (e), and (f)?
- Suppose we were to list all possible samples of $n = 20$ employee labels (but do not actually do this), then obtain the sample mean salary (\bar{x}) for each of these possible samples of size $n = 20$, and finally construct a histogram for these sample mean salaries. The histogram would display the sampling distribution of \bar{x} with simple random samples of size $n = 20$; if we were to calculate the value of the mean ($\mu_{\bar{x}}$) for this sampling distribution (but you do not need to actually do this!), what would we find the value to be?

(continued next page)

Self-Test Problem 14-2 - continued

- (i) How would the dispersion in the sampling distribution of \bar{x} with simple random samples of size $n = 20$ compare with the dispersion in the sampling distribution of \bar{x} with simple random samples of size $n = 4$?
- (j) How would the shape of the sampling distribution of \bar{x} with simple random samples of size $n = 20$ compare with the shape of the sampling distribution of \bar{x} with simple random samples of size $n = 4$?
- (k) Suppose a simple random sample of size n employees is to be selected, and the probability of obtaining a sample mean salary (\bar{x}) within 5 thousand dollars of the population mean ($\mu = 40$ thousand dollars) is to be calculated. Is this probability the same when $n = 4$ and when $n = 20$? (In other words, is the probability that \bar{x} is between 35 and 45 thousand dollars the same when $n = 4$ and when $n = 20$)? If yes, why; if no, in which case is the probability larger and why?

Answers to Self-Test Problems

- 14-1** (a) The sample consists of the 40 Econo brand light bulbs selected, the population of interest consists of all Econo brand light bulbs produced, the parameter of interest is the mean lifetime for all Econo brand light bulbs, and the statistic used to estimate the parameter is the mean lifetime for the sample of 40 Econo brand light bulbs selected. (b) The accessible population is probably not all Econo brand light bulbs produced, but consists only of those bulbs available at the time of sample selection. (c) Even though every Econo brand light bulb produced is not accessible, it is reasonable to assume that arbitrarily obtaining one bulb from those produced each day for 40 days in a row will give a random selection of bulbs.
- 14-2** (a) The histogram should display a height of 40% at 20 thousand dollars, a height of 30% at 40 thousand dollars, a height of 20% at 60 thousand dollars, and a height of 10% at 80 thousand dollars. (b) $\mu = 40$ thousand dollars. (c) The distribution of salaries is positively skewed. (d) $\mu_{\bar{x}} = \mu = 40$ thousand dollars. (e) The sampling distribution with $n = 4$ will show less dispersion than the distribution of salaries in the parent population. (f) The sampling distribution with $n = 4$ will be more symmetric and somewhat bell-shaped, while the distribution of salaries in the parent population is positively skewed. (g) The center of the distribution, the dispersion of the distribution, and the shape of the distribution should be consistent with the answers to parts (d), (e), and (f). (h) $\mu_{\bar{x}} = \mu = 40$ thousand dollars. (i) The sampling distribution with $n = 20$ will show less dispersion than the sampling distribution with $n = 4$. (j) The sampling distribution with $n = 20$ will be more symmetric and bell-shaped than the sampling distribution with $n = 4$. (k) The probability of obtaining a sample mean salary (\bar{x}) within 5 thousand dollars of the population mean $\mu = 40$ thousand dollars (i.e., \bar{x} between 35 and 45 thousand dollars) is larger when $n = 20$ than when $n = 4$. This is because as the sample size n increases, the sampling distribution of \bar{x} shows less dispersion around the population mean $\mu_{\bar{x}} = \mu = 40$ thousand dollars, which makes values closer to 40 thousand dollars more likely and values away from 40 thousand dollars less likely.

Summary

A *parameter* refers to a numerical summary corresponding to a population, and a *statistic* refers to a numerical summary corresponding to a sample. The symbol \bar{x} refers to a sample mean, which is a statistic; the Greek letter μ (mu) refers to a population mean, which is a parameter. The population from which the sample is to be selected can be called the *parent population*. The *sampling distribution of \bar{x}* based on simple random sampling is a frequency distribution displaying the possible values of \bar{x} together with the corresponding relative frequency of times each value occurs with simple random sampling. The symbol $\mu_{\bar{x}}$ refers to the mean of the population consisting of the possible values of \bar{x} from a given random sample size n , that is, the mean of the sampling distribution of \bar{x} with random samples of size n . When simple random samples of size n are selected from a parent population with mean μ and a finite variance, then the following three statements will be true:

- (1) $\mu_{\bar{x}} = \mu$.
- (2) The variation in the sampling distribution of \bar{x} is less than in the parent population and decreases as n increases.
- (3) The sampling distribution of \bar{x} becomes more bell-shaped decreases as n increases.