

# Unit 27

## One-Way Analysis of Variance

Objectives:

- To perform the hypothesis test in a one-way analysis of variance for comparing more than two population means

Recall that a two sample  $t$  test is applied when two population means are being compared with data consisting of two independent random samples. We discussed two available test statistics: the pooled  $t$  test statistic and the separate  $t$  test statistic. Our recommendation was that we just use the separate  $t$  test statistic, but as be seen from the formulas (displayed in an earlier unit and in Section B.2 of Appendix B, each of these test statistics is a difference between sample means  $\bar{x}_1 - \bar{x}_2$  divided by an appropriate standard error. We shall now consider a hypothesis test for comparing more than two population means with data consisting of independent random samples. We let  $k$  represent the number of populations; consequently, the data consists of  $k$  independent random samples, one from each population.

As an illustration, we shall consider Table 27-1 which displays data consisting of three random samples of rope breaking strengths (in pounds): one for rope brand Deluxe, one for rope brand Econ, and one for rope brand Nogood. Our interest is in a hypothesis test to see if there is any evidence of a difference in mean breaking strength among the rope brands. The null hypothesis of course states that all the means are equal. Since the null hypothesis in a two sample  $t$  test involves only two means, the alternative hypothesis could be one-sided or two-sided depending on whether we are looking for evidence of a difference in only one direction or in either direction. However, when the null hypothesis involves more than two means, there is only type of alternative hypothesis we consider: one stating that there is at least one difference among the means.

For instance, in comparing the mean breaking strength among the three rope brands, the null hypothesis states that the mean breaking strength is the same for the three rope brands. If this null hypothesis were not true, there are many different possibilities: the mean could be largest for Deluxe and equal for Econ and Nogood, the mean could be smallest for Deluxe and equal

for Econ and Nogood, the mean could be different for all three brands being largest for Deluxe and smallest for Nogood, the mean could be different for all three brands being largest for Deluxe and smallest for Econ, etc. The fact that so many possibilities exist is why the only alternative hypothesis we consider is the one stating that there is at least one difference among the means. As a result, a hypothesis test involving more than two means is always treated as one-sided.

In our illustration, the variable "Brand of Rope" is a qualitative variable whose different categories define the different populations being compared. In general, one may consider more than one qualitative variable, but we will not consider these more complex analyses here. When only one qualitative variable is used to defined the populations whose means are being compared, a hypothesis test can be derived from a technique known as a *one-way analysis of variance*, abbreviated as *one-way ANOVA*.

Analysis of variance may at first seem like a strange name for a procedure whose purpose is to compare means, but this name is actually very appropriate because the test statistic in an ANOVA is the ratio of two measures of variation: the variation between samples and the variation within samples. This test statistic is

<u>Deluxe</u>	165	162	159	162
<u>Econ</u>	156	163	158	
<u>Nogood</u>	151	154	160	

<u>Deluxe</u>	$n_D =$ _____	$\bar{x}_D =$ _____ pounds
<u>Econ</u>	$n_E =$ _____	$\bar{x}_E =$ _____ pounds
<u>Nogood</u>	$n_N =$ _____	$\bar{x}_N =$ _____ pounds

similar to a two sample  $t$  test statistic which has the difference between the two sample means as its numerator and a standard error as its denominator. We can think of the difference between means in the numerator as a measure of variation (i.e., the difference) between the two samples, and we can think of the standard error in the denominator as a measure of variation within samples.

The numerator of our one-way ANOVA test statistic with the data in Table 27-1 is going to measure how different from each other the three sample means are that is, the amount of variation among the sample means. It should be an easy task for you to calculate each of these sample means and complete Table 27-2 by entering the sample means and sample sizes. (You should find that  $n_D = 4$ ,  $\bar{x}_D = 162$ ,  $n_E = 3$ ,  $\bar{x}_E = 159$ ,  $n_N = 3$ ,  $\bar{x}_N = 155$ .)

The amount of variation among the sample means is measured from the deviations of the sample means from what we call the grand mean (similar to the way the sample variance measures the variation in a sample from the deviations of observations from the sample mean). This *grand mean*, denoted as  $\bar{x}_*$ , is the mean obtained when all samples are combined together into one sample; we represent this total sample size as  $n_*$ . You can easily check that when all  $n_* = 10$  observations of Table 27-1 are combined together, the grand mean is  $\bar{x}_* = 159$ . Variation among the sample means is based on squaring the deviation of each sample mean from the grand mean, multiplying by the corresponding sample size, and summing these results. This sum of squares is called the *between samples sum of squares*, which we shall denote as  $SSB$ . For the data of Table 27-1, you can easily check that  $SSB = 84$ .

The  $df$  (degrees of freedom) associated with this sum of squares for the numerator of our one-way ANOVA test statistic is one less than the number of samples being compared (i.e., one less than the number of categories of the qualitative variable which defines the populations in our one-way ANOVA). With  $k$  samples being compared, we have  $df = k - 1$  associated with  $SSB$ , which is  $3 - 1 = 2$  for the data of Table 27-1; we shall call this the *numerator  $df$*  for our test statistic. The numerator of our one-way ANOVA test statistic is

$$\frac{SSB}{(k - 1)} ,$$

which is called the *between samples mean square* and denoted as  $MSB$ . In general a *mean square* is a sum of squares divided by its associated degrees of freedom. For the data of Table 27-1,  $MSB = 84/2 = 42$ .

While the numerator of our one-way ANOVA test statistic measures how different from each other the  $k$  sample means are, the denominator of the test statistic measures how different from each other the observations within any particular sample are, that is, the amount of variation within any particular sample. This within sample variation is measured from the deviations of individual observations from the corresponding sample mean by squaring the deviation of each observation from its respective sample mean and summing these squares. This sum of squares is called the *within samples sum of squares* or the *error sum of squares* (or sometimes the *residual sum of squares*). We shall denote this sum of squares as  $SSE$ . For the data of Table 27-1, you can easily check that  $SSE = 86$ .

The  $df$  (degrees of freedom) associated with this sum of squares for the denominator of our one-way ANOVA test statistic is  $n_* - k$ . That is, we have  $df = n_* - k$  associated with  $SSE$ , which is  $10 - 3 = 7$  for the data of Table 27-1; we shall call this the *denominator  $df$*  for our test statistic. The denominator of our one-way ANOVA test statistic is

$$\frac{SSE}{(n_* - k)} ,$$

which is called the *within samples mean square* or the *error mean square* (or the *residual mean square*) and is denoted as  $MSE$ . For the data of Table 27-1,  $MSE = 86/7 = 12.286$ .

The formula for our one-way ANOVA test statistic is

$$\frac{MSB}{MSE} = \frac{SSB / (k - 1)}{SSE / (n_* - k)} .$$

For the data of Table 27-1, you should now easily be able to verify that the one-way ANOVA test statistic is 3.42. While we have outlined how this test statistic is calculated, a more detailed description of the calculation is provided in Section B.3 in Appendix B for the interested reader. By making use of the appropriate statistical software or programmable calculator, it is easy to obtain the sums of squares, degrees of freedom, mean squares, and test statistic value without having to grind through all the calculations. With data sets larger than that of Table 27-1, these calculations can be cumbersome.

This one-way ANOVA test statistic is similar to the pooled two sample  $t$  statistic in that the denominator measures variation within samples by assuming that the variation is roughly the same within the different samples. Recall that we recommended using the separate  $t$  statistic instead of the pooled  $t$  statistic to avoid having to be concerned about whether or not the assumption of the same variation within all samples is reasonable. While alternative test statistics are available for a one-way ANOVA for cases where samples show drastically different variation (and readily available with the appropriate statistical software or programmable calculator), we shall not discuss these in detail here. Our focus will be exclusively on the one-way ANOVA test statistic obtained by dividing the between samples mean square by the error mean square.

A two-sample  $t$  statistic is compared to the appropriate Student's  $t$  distribution in order to decide whether or not to reject the null hypothesis. We denote this test statistic by  $t(df)$  and refer to it as a  $t$  statistic with degrees of freedom  $df$ . In a one-way ANOVA, we decide whether or not to reject the null hypothesis by comparing our test statistic to an appropriate *Fisher's  $f$  distribution*. In order to perform the hypothesis test in a one-way ANOVA, we must first study the  $f$  distributions. Just as Table A.2 provides us with information about the standard normal distribution and Table A.3 provides us with information about the  $t$  distributions, Table A.4, which extends over several pages in Appendix A, provides us with information about the  $f$  distributions. The figures at the top of Tables A.2 and A.3 indicate that the standard normal distribution and the  $t$  distributions are each symmetric and bell-shaped. In contrast, the figure at the top of the first page of Table A.4 indicates that each  $f$  distribution is positively skewed. The figure also indicates that values from an  $f$  distribution are always nonnegative, unlike values from a normal distribution or a  $t$  distribution, which may be negative or positive.

Table A.4 contains information about several different  $f$  distributions. Each different  $t$  distribution is identified by  $df$  (degrees of freedom); however, each  $f$  distribution depends on two different  $df$ : one called *numerator degrees of freedom* and one called *denominator degrees of freedom*. Each column of Table A.4 is labeled by a value for numerator  $df$ , and each row is labeled by a value for denominator  $df$ . Corresponding to each combination of numerator  $df$  and denominator  $df$  is a set of four values. As indicated at the top of each page of Table A.4, the first value of the set is the  $f$ -score above which lies 0.10 of the area under the corresponding  $f$  density curve, the second value is the  $f$ -score above which lies 0.05 of the area under the corresponding  $f$  density curve, the third value is the  $f$ -score above which lies 0.01 of the area under the corresponding  $f$  density curve, and the fourth value is the  $f$ -score above which lies 0.001 of the area under the corresponding  $f$  density curve.

The notation we shall use to represent the  $f$ -scores in Table A.4 is similar to the notation we use to represent  $t$ -scores in Table A.3. For instance, just as we use  $t_{5,0.01}$  to represent the  $t$ -score above which lies 0.01 of the area under the density curve for the Student's  $t$  distribution with  $df = 5$ , we shall use  $f_{5, 13; 0.01}$  to represent the  $f$ -score above which lies 0.01 of the area under the density curve for the Fisher's  $f$  distribution with numerator  $df = 5$  and denominator  $df = 13$ . Notice that when designating  $f$ -scores in Table A.4, we include in the subscript both the numerator and denominator degrees of freedom followed by the area above the  $f$ -score.

Recall that our test statistic in a one-way ANOVA is  $MSB$  divided by  $MSE$ . As we have seen, the numerator  $df$  is  $k - 1$ , since  $MSB$  is obtained by dividing  $SSB$  by  $k - 1$ ; similarly, we have seen that the denominator  $df$  is  $n_* - k$ , since  $MSE$  is obtained by dividing  $SSE$  by  $n_* - k$ . Consequently, we shall use

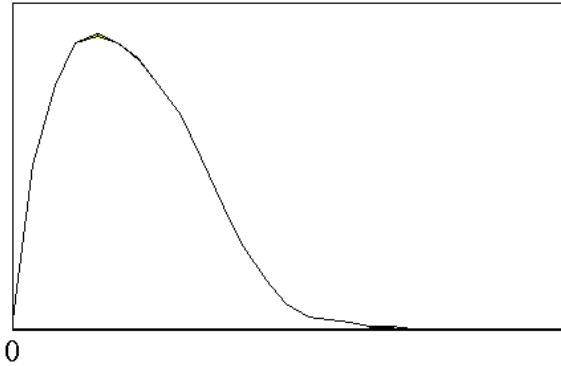
$f_{n_* - k, k - 1}$  to denote this test statistic; that is, we can write

$$f_{n_* - k, k - 1} = \frac{MSB}{MSE} = \frac{SSB / (k - 1)}{SSE / (n_* - k)},$$

which we can refer to as Fisher's  $f$  statistic with  $k - 1$  numerator degrees of freedom and  $n_* - k$  denominator degrees of freedom. Recall that for the data of Table 27-1, we have found that  $f_{2,7} = 3.42$ .

Our null hypothesis in a one-way ANOVA states that the population means are all equal. If all sample means turned out to be equal to each other, the  $f$  statistic would be equal to zero (since  $SSB$  would be equal to zero). However, even if the null hypothesis in a one-way ANOVA is true, we still expect there to be some random variation among the sample means. The figure at the top of the first page of Table A.4 illustrates the distribution of the  $f$  test statistic if the null hypothesis is true. Figure 27-1 displays this same typical density curve for an  $f$  distribution. Notice that the shape of this density curve suggests that a high proportion of the  $f$ -scores for most  $f$  distributions are roughly speaking in the range from 0 to 2. In a one-way ANOVA, an  $f$  test statistic which is unusually large would provide us with evidence that at least one population mean is different from the others. We then define the rejection region in a one-way ANOVA by the  $f$ -score above which lies  $\alpha$  of the area under the corresponding density curve, where  $\alpha$  is of course the significance level. The shaded area in the figure at the top of the first page of Table A.4 graphically illustrates the type of rejection region we use in a one-way ANOVA.

**Figure 27-1**  
**Typical Density Curve for**  
**a Fisher's  $f$  Distribution**



The same four steps we have used in the past to perform hypothesis tests are used in a one-way ANOVA. Let us use a one-way ANOVA with the data of Table 27-1 to see if there is any evidence of a difference in mean breaking strength among the rope brands Deluxe, Econ, and Nogood. We shall choose a 0.01 significance level for the one-way ANOVA.

The first step is to state the null and alternative hypotheses, and choose a significance level. The null hypothesis of course states that mean breaking strength is equal for the rope brands Deluxe, Econ, and Nogood. We can state this null hypothesis in an abbreviated form and complete the first step of the hypothesis test as follows:

$$H_0: \mu_D = \mu_E = \mu_N \text{ vs. } H_1: \text{At least one of } \mu_D, \mu_E, \text{ and } \mu_N \text{ is different } (\alpha = 0.01).$$

We complete the second step of the hypothesis test by calculating the  $f$  test statistic. Recall that we did this earlier and found that

$$f_{2,7} = 3.42.$$

The third step is to define the rejection region, decide whether or not to reject the null hypothesis, and obtain the  $p$ -value of the test. As we have indicated earlier, our rejection region is defined by the  $f$ -score above which lies an area equal to the significance level. For this hypothesis test, the rejection region is defined by the  $f$ -score above which lies 0.01 of the area under the  $f$  density curve with 2 numerator degrees of freedom and 7 denominator degrees of freedom; from Table A.4, we see that this  $f$ -score is  $f_{2,7;0.01} = 9.55$ . We can then define our rejection region algebraically as

$$f_{2,7} \geq 9.55.$$

Graphically, we can picture the rejection region as corresponding to the shaded area in the figure at the top of the first page of Table A.4, where the value on the horizontal axis defining the rejection region is  $f_{2,7;0.01} = 9.55$ . Since our test statistic  $f_{2,7} = 3.42$  is not in the rejection region, our decision is not to reject  $H_0: \mu_D = \mu_E = \mu_N$ ; in other words, our data does not provide sufficient evidence against  $H_0: \mu_D = \mu_E = \mu_N$ .

The  $p$ -value of this hypothesis test is the probability of obtaining a test statistic value  $f_{2,7}$  which represents greater differences among the sample means than does the value actually observed  $f_{2,7} = 3.42$ . Graphically, we can picture the  $p$ -value of this hypothesis test as corresponding to the shaded area in the figure at the top of the first page of Table A.4, where the value on the horizontal axis is our observed test statistic value  $f_{2,7} = 3.42$ . By looking at the entries of Table A.4 corresponding to 2 numerator degrees of freedom and 7 denominator degrees of freedom, we find that the observed test statistic  $f_{2,7} = 3.42$  is between  $f_{2,7;0.10} = 3.26$  and  $f_{2,7;0.05} = 4.74$ . This tells us that the area above  $f_{2,7} = 3.42$ , which is the  $p$ -value, is between 0.05 and 0.10. We indicate this by writing  $0.05 < p\text{-value} < 0.10$ . The fact that  $0.05 < p\text{-value} < 0.10$  confirms to us that  $H_0$  is not rejected with  $\alpha = 0.01$ . However, this also tells us that  $H_0$  would be rejected with  $\alpha = 0.10$ , but would not be rejected with  $\alpha = 0.05$ .

To complete the fourth step of the hypothesis test, we can summarize the results of the hypothesis test as follows:

Since  $f_{2,7} = 3.42$  and  $f_{2,7;0.01} = 9.55$ , we do not have sufficient evidence to reject  $H_0$ . We conclude that the mean breaking strength is not different for the Deluxe, Econ, and Nogood rope brands ( $0.05 < p\text{-value} < 0.10$ ). Since  $H_0$  is not rejected, no further analysis is necessary.

Notice that in the statement of results, the last sentence indicates that no further analysis is necessary. This is always true in a one-way ANOVA when we do not reject the null hypothesis, since concluding that the population means are equal leaves us with no unanswered questions. On the other hand, whenever we reject the null hypothesis in a one-way ANOVA, we are concluding that at least one of the population means is different, and this leaves us with the question of which population mean or means are different. Answering this question requires further analysis. One type of analysis available to identify significantly different population means is discussed in the next unit.

**Table 27-3**  
**One-Way ANOVA Table**

<i>Source</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>f</i>	<i>p-value</i>
between	<i>SSB</i>	$k - 1$	<i>MSB</i>	$f_{k-1, n_* - k}$	
Error	<i>SSE</i>	$n_* - k$	<i>MSE</i>		
Total	<i>SST</i>	$n_* - 1$			

Often, the results from an ANOVA are summarized in an ANOVA table. Table 27-3 displays how a *one-way ANOVA table* is organized. The first column is labeled *Source*. This refers to the three sources of variation on which the one-way ANOVA is based: the variation between samples, the variation within samples, and the total variation. We can think of the total variation as consisting of two components: the variation between samples and the variation within samples. This is because  $SSB + SSE = SST$ . The second and third columns of the ANOVA table are respectively labeled *SS* and *df*, referring to sums of squares and degrees of freedom. Notice that the total degrees of freedom is one less than the total sample size, which is the sum of the between samples degrees of freedom and the error degrees of freedom. The last three columns of the ANOVA table contain respectively the two mean squares, the  $f$  statistic and the  $p$ -value.

**Table 27-4**  
**One-Way ANOVA Table for the Data of Table 27-1**

<i>Source</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>f</i>	<i>p-value</i>
Rope Brands					
Error					
Total					

Table 27-4 is the one-way ANOVA table for the data of Table 27-1. Notice that in place of the label "Between" we have put the label "Rope Brands" to emphasize that the three different brands of rope were being compared in this one-way ANOVA. Using the calculations we have done previously for the data of Table 27-1, complete the ANOVA table in Table 27-4. (After you complete the ANOVA table, you should have 84, 86, and 170 respectively in the *SS* column; you should have 2, 7, and 9 in the *df* column; you should have 42 and 12.286 respectively in the *MS* column; you should have 3.42 in the *f* column; you should have either  $0.05 < p\text{-value} < 0.10$  or an exact *p-value*, obtained from a calculator or computer, in the *p-value* column.)

Even though the one-way ANOVA is based the assumption that independent random samples are selected from normally distributed populations all having the same standard deviation, the *f* test in the one-way ANOVA is known to be somewhat *robust*, which means that it will still perform well in several situations when the assumptions are not satisfied. In general, it is advisable to check how far data departures from the necessary assumptions in order to make a decision as to whether or not to employ an alternative to the one-way ANOVA.

**Self-Test Problem 27-1.** A 0.01 significance level is chosen for a hypothesis test to see if there is any evidence of a difference in mean yearly income among different political party affiliations in a particular state. The individuals selected for the SURVEY DATA, displayed as Set 1-1 at the end of Unit 1, are treated as comprising four random samples: one from Republicans, one from Democrats, one from Independents, and one from Others.

- (a) Explain how the data for this hypothesis test is appropriate for a one-way ANOVA.
- (b) Complete the four steps of the hypothesis test by completing the table titled *Hypothesis Test for Self-Test Problem 27-1*. As part of the second step, complete the construction of the one-way ANOVA table, displayed as Table 27-5, where you should find that  $SSB = 183.825$ ,  $SSE = 7143.375$ , and  $f_{3, 26} = 0.22$ .
- (c) Decide which of the following would be best as a graphical display for the data and say why: (i) four pie charts, (ii) four scatter plots, (iii) four box plots.
- (d) Considering the results of the hypothesis test, decide which of the Type I or Type II errors is possible, and describe this error.
- (e) Decide whether  $H_0$  would have been rejected or would not have been rejected with each of the following significance levels: (i)  $\alpha = 0.05$ , (ii)  $\alpha = 0.10$ .
- (f) What would the presence of one or more outliers in the data suggest about using the *f* statistic?

**Hypothesis Test for Self Test Problem 27-1**

Step 1  $H_0$ :  $H_1$ :  $\alpha =$

Step 2

Step 3

Step 4

**Table 27-5**

**One-Way ANOVA Table for Self-Test Problem 27-1**

<i>Source</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>f</i>	<i>p-value</i>
Error					
Total					

### Answers to Self-Test Problems

- 27-1** (a) The data consists of independent random samples selected from more than two populations, and the purpose of a one-way ANOVA is to compare means of such data.
- (b) Step 1:  $H_0: \mu_R = \mu_D = \mu_I = \mu_O$ ,  $H_1$ : At least one of  $\mu_R$ ,  $\mu_D$ ,  $\mu_I$ , and  $\mu_O$  is different, ( $\alpha = 0.01$ )
- Step 2:  $n_R = 10$ ,  $\bar{x}_R = 44.5$ ,  $n_D = 8$ ,  $\bar{x}_D = 49.4$ ,  $n_I = 4$ ,  $\bar{x}_I = 42.5$ ,  $n_O = 8$ ,  $\bar{x}_O = 44.0$ ,  $f_{3, 26} = 0.22$ .
- Step 3: The rejection is  $f_{3, 26} \geq 4.64$ .  $H_0$  is not rejected;  $0.10 < p$ -value.
- Step 4: Since  $f_{3, 26} = 0.22$  and  $f_{3, 26; 0.01} = 4.64$ , we do not have sufficient evidence to reject  $H_0$ . We conclude that there is no difference in mean yearly income among the four political party affiliations in the state ( $0.10 < p$ -value). Since  $H_0$  is not rejected, no further analysis is necessary.
- In Table 27-5, the first label in the *Source* column should be “Party Affiliation”; the entries in the *SS* column should respectively be 183.825, 7143.375, and 7327.200; the entries in the *df* column should respectively be 3, 26, and 29; the entries in the *MS* column should respectively be 61.275 and 274.7452; the entry in the *f* column should be 0.22; the entry in the *p*-value column should be either  $0.10 < p$ -value or an exact *p*-value, obtained from a calculator or computer.
- (c) Since yearly income is a quantitative variable, four box plots, one for each party affiliation category, is an appropriate graphical display.
- (d) Since  $H_0$  is not rejected, the Type II error is possible, which is concluding that the means are all equal when in reality at least one mean is different.
- (e)  $H_0$  would not have been rejected with both  $\alpha = 0.05$  and  $\alpha = 0.10$ .
- (f) The presence of one or more outliers in the data would suggest that the *f* statistic may not be appropriate.

### Summary

A two sample *t* test statistic is the difference between sample means  $\bar{x}_1 - \bar{x}_2$  divided by an appropriate standard error. A hypothesis test for comparing  $k > 2$  population means with data consisting of independent random samples is available from the *one-way analysis of variance (ANOVA)*. The test statistic in the one-way ANOVA is somewhat similar to a two sample *t* test statistic, in the sense that it is the ratio of two measures of variation: the variation between samples and the variation within samples.

The *grand mean* denoted as  $\bar{x}_*$ , is the mean obtained when all samples are combined together into one sample; we represent this total sample size as  $n_*$ . The squared deviation of each sample mean from the grand mean is multiplied by the corresponding sample size, and these results are summed to produce what is called the *between samples sum of squares*, abbreviated as *SSB*. We divide *SSB* by  $k - 1$  to obtain the *between samples mean square*, abbreviated as *MSB*. The *MSB* is the numerator of the test statistic in the one-way ANOVA and is a measure of the variation between samples.

The deviation of each observation from its respective sample mean is squared, and these results are summed to produce what is called the *within samples sum of squares* or the *error sum of squares* (or sometimes the *residual sum of squares*), abbreviated as *SSE*. We divide *SSE* by  $n_* - k$  to obtain the *within samples mean square* or the *error mean square* (or the *residual mean square*), abbreviated as *MSE*. The *MSE* is the denominator of the test statistic in the one-way ANOVA and is a measure of how different from each other the observations within any particular sample are, that is, the amount of variation within any particular sample.

In general, a *mean square* is a sum of squares divided by its associated degrees of freedom. The test statistic in a one-way ANOVA is the ratio of the mean square *MSB* divided by the mean square *MSE*. This test statistic is called a Fisher’s *f* statistic. If the sample means were all equal, this test statistic would be equal to

zero, since the numerator  $MSB$  would be zero. The null hypothesis in a one-way ANOVA states that the population means are all equal and is rejected when the Fisher's  $f$  statistic

$$f_{n_* - k, k - 1} = \frac{MSB}{MSE} = \frac{SSB / (k - 1)}{SSE / (n_* - k)}$$

is sufficiently large, when compared to the Fisher's  $f$  distribution with  $k - 1$  numerator degrees of freedom and  $n_* - k$  denominator degrees of freedom. The results from a one-way ANOVA are often organized into the one-way ANOVA table displayed as Table 27-3.

The one-way ANOVA is based the assumption that independent random samples are selected from normally distributed populations all having the same standard deviation. The  $f$  test in the one-way ANOVA is known to be somewhat *robust*, which means that it will still perform well in several situations when the assumptions are not satisfied. Alternative tests are available when there is uncertainty about whether one or more of the assumptions are not satisfied. Access to the appropriate statistical software or programmable calculator allows one to avoid having to do much of the calculation necessary in a one-way ANOVA.