Some Well-Known Sums

$$1 + 2 + 3 + \cdots + m = \frac{m(m+1)}{2}$$

for any positive integer *m*

$$1^2 + 2^2 + 3^2 + \cdots + m^2 = \frac{m(m+1)(2m+1)}{6}$$

for any positive integer *m*

$$1^3 + 2^3 + 3^3 + \cdots + m^3 = \frac{m^2(m+1)^2}{4}$$

for any positive integer *m*

$$(a+b)^m = \sum_{k=0}^{m} \binom{m}{k} a^{m-k} b^k$$

for any positive integer *m* and any real numbers *a* and *b*

$$1 + w + w^2 + w^3 + \cdots + w^m = \frac{1-w^{m+1}}{1-w}$$

for any positive integer *m*

$$\frac{1}{(1)(2)} + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \cdots + \frac{1}{(m)(m+1)} = \frac{m}{m+1}$$

for any positive integer *m*

Some Well-Known Series

$$e^{w} = 1 + w + \frac{w^2}{2!} + \frac{w^3}{3!} + \frac{w^4}{4!} + \cdots \quad \text{for } -\infty < w < \infty$$

$$\frac{1}{1-w} = 1 + w + w^2 + w^3 + w^4 + \cdots \quad \text{for } -1 < w < 1$$

$$\frac{1}{(1-w)^2} = 1 + 2w + 3w^2 + 4w^3 + \cdots \quad \text{for } -1 < w < 1$$

$$\frac{1}{(1-w)^3} = 1 + \binom{3}{2} w + \binom{4}{2} w^2 + \cdots \quad \text{for } -1 < w < 1$$

$$\frac{1}{(1-w)^m} =$$

$$\binom{m-1}{m-1} + \binom{m}{m-1} w + \binom{m+1}{m-1} w^2 + \binom{m+2}{m-1} w^3 +$$

$$\text{for } -1 < w < 1$$

$$\ln(w) =$$

$$(w-1) - \frac{1}{2}(w-1)^2 + \frac{1}{3}(w-1)^3 - \frac{1}{4}(w-1)^4 + \frac{1}{5}(w-1)^5 - \cdots$$

$$\text{for } 0 < w \le 2$$

or

$$\frac{w-1}{w} + \frac{1}{2}\left(\frac{w-1}{w}\right)^2 + \frac{1}{3}\left(\frac{w-1}{w}\right)^3 + \frac{1}{4}\left(\frac{w-1}{w}\right)^4 + \frac{1}{5}\left(\frac{w-1}{w}\right)^5 + \cdots$$

$$\text{for } 1/2 < w$$