

# The Optimal Mix of Traditional and Roth Retirement Accounts with Progressive Taxation

By

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## Abstract

U.S tax law allows households to use traditional tax-deferred retirement accounts and Roth tax-free retirement accounts. The conventional wisdom is that the traditional account should be used by households that expect their tax rate to fall in retirement, while the Roth accounts should be used by households that expect their tax rate to rise in retirement. This paper moves beyond this conventional wisdom by modeling households that can affect both their working and retirement tax rates through their use of both traditional and Roth accounts. Simultaneous use of both traditional and Roth accounts is an equilibrium solution. The ability of households to choose between retirement account types not only aids tax minimization for them, but it also improves income smoothing and horizontal equity for households with year-to-year income variability.

Keywords: Roth, 401(k), Retirement, IRA

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## 1. Introduction

Individual Retirement Accounts (IRAs) and 401(k) accounts are well known U.S. vehicles for people to accumulate retirement savings in a tax-sheltered environment. More recently, Roth IRAs entered the scene in 1998, and those were followed in 2006 with Roth 401(k) accounts (Novack, 2006). The traditional IRA and 401(k) accounts allow deposits to be deducted from taxable income in the year of the deposit and then proceeds are added to taxable income when withdrawn during retirement. The Roth accounts are funded with post-tax dollars, but then they do not count as taxable income when withdrawn during retirement. Thus, each type of account allows for a desirable but different type of tax savings. The combined annual limits per worker for these accounts are currently (2008) \$5,000 for IRA accounts and \$15,500 for 401(k) accounts<sup>1</sup> (Internal Revenue Service, 2007 [1] and [2]).

For people who expect their marginal tax bracket “ $t$ ” to be the same before and during retirement, these two accounts offer identical tax advantages. This is easily seen by comparing \$1 invested in a traditional, deductible account, which is shown on the left of the following equation. The \$1 is untaxed and can be invested at rate  $R$ , then during retirement (period 2) it is taxed, leaving  $(1-t_2)$  percentage of it for retirement consumption. The right hand side of the equation shows the same \$1 used for a Roth account. Here the dollar is immediately taxed in period 1, leaving  $(1-t_1)$  of it to go into the account, which then is invested at rate  $R$ :

$$(1) \quad \$1(1+R)(1-t_2) = \$1(1-t_1)(1+R).$$

They are clearly identical for our case where  $t_1 = t_2$ . A difference arises when  $t_1$  differs from  $t_2$ , so that the  $=$  becomes a  $>$  ( $<$ ) when  $t_1 >$  ( $<$ )  $t_2$ . This observation leads to the well publicized rule of thumb that one should use traditional, deductible accounts when one’s working tax rate is higher than the expected

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<sup>1</sup> For those over 50, the limits are higher. Higher income households are not eligible for IRA accounts, but there are currently no income limits on 401(k) or the similar 403(b) accounts used widely in U.S. universities and research institutions.

retirement tax rate, while Roth accounts are better when one's tax rate is expected to rise in retirement.

While marketing and promotion of Roth style accounts is common (See for example *In the Vanguard*, 2007), the academic literature on the distinction between traditional and Roth style accounts has been fairly limited. Some of this literature focuses on the nuts and bolts of how they differ, such as Adelman and Cross (2006). Other articles analyze specific conditions under which one or the other type of account is best (Burman, Gale and Weiner (2001) and Butterfield, Jacobs and Larkins (2000)), while Gokhale, Kotlikoff and Neumann (2001) run simulations to show how ordinary use of standard 401(k) accounts can ironically increase lifetime taxes. A simulation study by Kotlikoff, Marx and Rapson (2008), looks at not only the impact of different account types on lifetime standards of living, but also the impact of various tax reforms. Most articles point out the mathematical equivalence between the two types of accounts for people with steady marginal tax rates. Burman, Gale and Weiner (2001) and Caliendo and Lewis (2004) both make the point that the Roth is preferred for people saving the maximum legal annual contribution limit. That is because Roth accounts require tax payment immediately, therefore making a \$5,000 Roth deposit (the current IRA maximum) effectively larger than funding a \$5,000 traditional IRA, because the traditional account will be taxed at withdrawal, while the Roth account will not be. Funding a \$5,000 Roth account then requires more dollars of income but also allows more dollars to be tax sheltered.

Literature on optimal joint use of traditional and Roth accounts is scarcer still. Johnson (2003) analyzes optimal portfolio allocation between traditional and Roth accounts, with a focus on how concentrating different types of assets in each type of account can improve post-tax outcomes. Burman, Gale and Weiner (2001) and Adelman and Cross (2005) both provide examples of how a person's optimal choice of account type often has a lifecycle component, with Roth accounts being preferred for younger, lower income workers and traditional accounts better for older, higher income people. Thus, what is the best option for a 25 year old may change when he or she is 45.

This paper explicitly models the optimal mix between traditional and Roth accounts that a net worth maximizing saver would employ. Significant and regular deposits to retirement accounts can and do affect household tax brackets, both before and during retirement. The model analyzes the optimal retirement account strategy when the very act of depositing into retirement accounts alters working and retirement marginal tax rates. Under this very real scenario of endogenously-determined marginal tax rates, it often becomes optimal to fund both traditional and Roth accounts simultaneously, rather than just choosing one or the other. The remainder of the paper is organized as follows. Section 2 develops the model and solves for the optimal mix between traditional and Roth retirement accounts. Section 3 considers extensions and refinements to the general result, as well as the policy implication of allowing household choice across account types. Section 4 concludes.

## 2. The Model

For a benchmark case, consider a two period model in which a person earns income “ $T$ ” in period 1 and is retired in period 2. Retirement income comes only from period 1 savings of “ $S$ ,” and that saving can either be through a traditional retirement account (“traditional account”) or a Roth retirement account (“Roth account”), which could take the form of an IRA, a 401(k), a 403(b) or any other retirement account that can take on the traditional or Roth properties. Traditional account deposits are subtracted from period 1 taxable income. During retirement, funds withdrawn from that account along with its investment income are then taxed as period 2 income. If the period 1 savings are put into a Roth account, they are not subtracted from period 1 taxable income. However, the Roth account and its investment earnings are not counted as income when withdrawn during retirement.

Taxes are progressive in each period “ $k$ ,” with a continuously rising marginal tax rate, and the tax rate structure is assumed to be constant across periods:

$$(2) \quad t_k = bm_k,$$

where  $b$  = a progressivity parameter

$m_k$  = taxable income in period  $k$

$t_k < 1$ .

Real world tax rates are not continuously progressive, but this functional form allows for standard calculus techniques to be used. The use of parameter “ $b$ ” to convert income “ $m$ ” into a tax rate is a very straightforward method of modeling a continuously progressive marginal tax rate so that tax rate  $t$  rises in lock step with income. A more complicated form would not change the fundamental results. Section 3.2 discusses the impact of the discrete tax brackets that are currently used. The restriction of  $t_k < 1$  is made so that this model will only consider the reasonable cases of marginal tax rates less than 100%. After-tax income in a particular period is the integral of  $(1-t_k)$  across all income, evaluated at the taxable income. Hence, in period 1, with gross income  $I$  and traditional account deposit “ $A$ ,” taxable income is  $I - A$ , and with the use of (2), after-tax income is:

$$(3) \quad m_1(A) = \int_{m_1=0}^{I-A} (1 - bm_1) dm_1.$$

The traditional account earns interest at rate “ $R$ ,” and this creates after-tax income in retirement of:

$$(4) \quad m_2(A) = \int_{m_2=0}^{A(1+R)} (1 - bm_2) dm_2.$$

Roth account deposits are determined by subtracting traditional account deposits  $A$  from the total savings budget  $S$  and then paying period 1 taxes on that difference before investing the remainder in the Roth account. Thus, the Roth account deposit will be:

$$(5) \quad \int_{m_1=I-S}^{I-A} (1 - bm_1) dm_1.$$

The after-tax remainder of the savings budget is used here to make a fair comparison between deposits to the two accounts. Were the entire  $S - A$  value used, the Roth account would artificially inflate total savings, because its taxes would be prepaid by funds outside of  $S$ , while we have defined the traditional account to pay taxes in period 2 out of its balance. Note that the taxes are computed from the upper marginal rates of  $(I-S)$  through  $(I-A)$  of income, because that is the marginal income on which taxes would be avoided if the Roth money would instead be used to increase traditional account deposit  $A$ . This account can be invested at rate  $R$  to earn tax-free income in retirement:

$$(6) \quad \int_{m_1=I-S}^{I-A} (1 - bm_1) dm_1 (1+R).$$

The household problem is to maximize lifetime wealth “ $W$ ” by choosing the optimal mix of traditional and Roth 401(k) accounts. This lifetime wealth then is the sum of and first period after-tax income, less retirement account deposits, and retirement after-tax income, which is  $(3) - (5) + (4) + (6)$ :

$$(7) \quad W(A) = \int_{m_1=0}^{I-A} (1 - bm_1) dm_1 - \int_{m_1=I-S}^{I-A} (1 - bm_1) dm_1 \\ + \int_{m_2=0}^{A(1+R)} (1 - bm_2) dm_2 + \int_{m_1=I-S}^{I-A} (1 - bm_1) dm_1 (1+R).$$

Note that this function presumes a zero inflation rate, so that the marginal tax rate functions are the same in each period. Time preference of money is also not an issue, since regardless of the mix between traditional and Roth accounts, the household is diverting  $S$  dollars of income from first period consumption. If a household wants to spend more in period 1, it could borrow against retirement income.

The central conclusion from this model can now be summarized by Proposition 1.

*Proposition 1.* When a person can control the realization of taxable income across working and retirement periods using traditional and Roth retirement accounts, and when marginal tax rates are continuously progressive, then the lifetime wealth-maximizing retirement account strategy is to deposit into the traditional account until marginal tax rates are equalized across years, then to fund Roth accounts out of any remaining funds allocated for savings.

*Proof.* Roth and traditional account deposits are complements in the savings budget  $S$ , so we can determine the optimal mix via the first order condition of  $W$  with respect to  $A$ :

$$(8) \quad \frac{\partial W(A)}{\partial A} = b(I - A) + R - bA(1 + R)^2 - (1 - b(I - A))R = 0.$$

Solve for  $A$  to get the optimal level of traditional account use:

$$(9) \quad A^* = \frac{I}{2 + R}.$$

We can substitute  $A^*$  back into (3) and (4) to see that this optimal choice of  $A$  equalizes taxable income  $m$  across both periods and thus also equalizes marginal tax rate  $bm$  across periods.:

$$(10) \quad m_1(A^*) - m_2(A^*) = \int_{m_1=0}^{I-A^*} (1 - bm_1) dm_1 - \int_{m_2=0}^{A^*(1+R)} (1 - bm_2) dm_2 = 0.$$

Note that the choice of  $A^*$  is independent of total savings level  $S$ . This implies that once the traditional account deposit has been increased to equalize marginal tax rates across periods, all remaining savings should go into a Roth account,

which has no impact on marginal tax rates in either period. Should  $A^*$  be greater than  $S$ , then the choice of  $A$  moves to a corner solution of  $A = S$  without equalizing the marginal tax rates, and retirement income and tax rates remain lower than working period values. Hence the fact that the Roth account gets a zero deposit at this corner solution is consistent with the principle that Roth accounts are not appropriate for people whose retirement tax bracket will be lower than their working years' tax bracket. QED

Under the  $A^*$  solution, all gains of tax deferral from traditional accounts have been exploited and further additions would push the retirement tax rate to a higher level than the working period tax rate. Returning again to the conventional wisdom, if the retirement marginal tax rate is higher than the working marginal tax rate, then Roth accounts become best. With our fixed savings budget, depositing more into a Roth account lowers the traditional account deposit and that action will push the tax rates back toward the equality of marginal tax rates occurring at  $A^*$ .

### 3. Discussion and Extensions

The intuition that follows from Proposition 1 of Section 2 is quite general and can be applied to various extensions that customize the model to different household expectations, time horizons and policy scenarios. These extensions are addressed in this section.

#### 3.1 Expected Tax Rate Changes

So far this model has assumed that the progressive tax rate system will stay constant across periods, but many people have expectations that the tax system will shift in the future. This assumption can be relaxed in the model by assuming that taxes are progressively scaled by parameter “ $c$ ” in the retirement period 2 with perfect foresight:

$$(11) \quad t_2 = cm_2,$$

with  $c$  possibly different than parameter  $b$  in (2). The amended taxable retirement income previously described by (4) becomes:

$$(12) \quad m_2(A) = \int_{m_2=0}^{A(1+R)} (1 - cm_2) dm_2.$$

Using (12) instead of (4) in wealth function (7) leads to a new form for the optimal period 1 traditional account deposit,  $A^*$ :

$$(13) \quad A^*(b,c) = \frac{bI}{c(1+R) + b}.$$

It is easy to see that (13) reverts back to (9) when  $c = b$ . When  $c > b$ , which means that future taxes will be more progressive and higher for any given taxable income, then (13) becomes smaller than (8). This means that higher future tax rates will induce lower working period traditional account deposits and higher Roth deposits, as realizing more taxable income in the working period becomes a way to avoid higher retirement period taxes. Similar logic implies that if  $c < b$ , then optimal traditional account deposits will rise. In other words, tax deferment is less valuable when the income must be later realized at higher tax rates, while the tax deferment becomes more useful when the income can be deferred until tax rates fall.

If the expected future is one of a flat rate tax of rate  $f$ , then for any future taxable income, (2) becomes:

$$(14) \quad t_2 = f.$$

Now the amended taxable retirement income is:

$$(15) \quad m_2(A) = \int_{m_2=0}^{A(1+R)} (1 - f) dm_2,$$

and replacing (4) with (15) leads to an optimal period 1 traditional account deposit of:

$$(16) \quad A^*(b,f) = \frac{bI - f}{b}.$$

If we evaluate  $t_1$  at  $m(A^*(b,f))$ , then  $t_1 = f$ . In other words, the interior solution is to use the traditional account to move the working period marginal tax rate to the future period flat rate, with the balance of savings going to Roth accounts.

### 3.2 Tax Bracket Systems

The current U.S. tax code is progressive, but not continuously progressive like the model in this paper. Rather, tax rates make discreet jumps and then stay constant for tens of thousands of dollars at a time. Thus, a family of four in the U.S. with income of \$100,000 and taking the standard deduction is currently subject to 0%, 10%, 15% and 25% brackets (Internal Revenue Service, 2007 [3]). How does this system fit in with the theory presented in this paper? The same basic logic holds: use traditional accounts to lower working period income and tax rates and shifting that income forward to increase retirement period income and tax rates until the tax rates are equal. Once, they are equal, then the traditional and the Roth accounts are mathematically equal in their after-tax returns (as we saw in equation (1)), so a household would be indifferent between the mix, so long as the tax brackets can't be further altered. If this is pushed to the changing of a bracket, then any further increases to savings should be put into the Roth account. An exception to this indifference occurs when the tax payer is at a corner solution on total deposits for the year, which is to say the total preferred savings  $S$  is greater than the legal account maximum deposit limits. In this case, the Roth account represents more possible after-tax savings, since the tax payments are paid with money that does not count in the 401(k) or IRA limit. This mirrors the Caliendo and Lewis (2004) result.

### 3.3 Multiple periods

The model deals with only two periods, one earning and both of them paying taxes and spending. What about multiple periods of either earnings or retirement? In the case of multiple periods of earnings, one can update the model to include two or more earnings periods by simply using two or more working-period, after-tax incomes from (3) when defining wealth  $W$  and then scaling them with an extra year or years of investment earnings. This exercise is demonstrated in the appendix with a two period earning model added to create a three period model. The result is that traditional account deposits are used to equate marginal tax rates in all three periods, with any further desired savings entering Roth accounts. So, the number of choices increases, but the basic result of using the accounts to equalize marginal tax rates is robust.

Multiple earnings periods does bring about the possibility of another corner solution in which only a Roth account is funded during certain years. This will occur when at least one of the earnings periods has a low enough income so that the marginal tax rate for the year is lower than the tax rate for the other earnings years and retirement. Then, the household will not want to further lower the low marginal rate year with a traditional account, while deferring the income to another higher tax rate year. Hence, any savings will go into a Roth that will lower the retirement tax rate. For a person following a traditional career path of rising working years income, this would mean that Roth accounts are more likely to be preferred at the beginning of the career, with traditional accounts entering the mix as marginal tax rates increase with income increases.

Multiple retirement periods can also be easily added. Maximizing the after-tax retirement income with multiple periods involves equalizing the taxable, traditional account withdrawals across years. That assumes that smooth consumption is either preferred, or that a different pattern of expenditures could be obtained by borrowing against the wealth maximizing pattern of equalized distributions. The portion of retirement savings in Roth accounts could also allow for a lumpier consumption pattern, since distribution from the Roth accounts have zero tax consequences.

### 3.4 Reversing the Process with IRA Conversions and Recharacterizations

The principle of using accounts to minimize taxes via income smoothing can become even more exact with the use of IRA conversions. Taxpayers who fall below specific income limits are able to convert existing traditional IRAs into Roth IRAs. When this is done, the amount of the conversion is added to taxable income that year. This allows a person who has existing IRAs to realize extra taxable income during lower earnings and tax rate years, rather than waiting until a higher tax rate retirement to withdraw and realize the income. The exact monetary quantity of the conversion can be used to move the current marginal tax rate up to the interior solution marginal tax rate. This conversion can even be reversed (called a “recharacterization”) for fine tuning taxable income any time before the tax filing deadline (Blankenship, 2005). This means fewer corner solutions with unequal marginal tax rates during low income years for people with variable incomes.

### 3.5 A Numerical Example

For a basic numerical example, suppose that after all deductions except for retirement account funding, a household earns annual taxable income of \$80,000 plus or minus \$5,000. It wants to use \$15,000 per year to create a flow of income in retirement. After computing the number of years to work and the expected number of retirement years, this household determines that it can save \$10,000 per year to generate a stream of \$70,000 per year in taxable retirement income<sup>2</sup>. In keeping with the model of the paper, there is zero inflation. Each working year, it will want to lower taxable income to \$70,000 and then put the after-tax balance of the \$15,000 into a Roth account. In year one, it earns \$80,000, so it puts \$10,000 into the traditional and the net-of-taxes value of its last \$5,000 of income into the Roth account. In year two it earns \$75,000, and it puts \$5,000 into the traditional and the \$10,000, less taxes, into the Roth. In year three it earns \$85,000, so it puts all \$15,000 into the traditional account. During retirement, \$70,000 is withdrawn each year from the traditional account, and the Roth money can be spent without tax consequence. If this generates too much income in retirement, the tax gains

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<sup>2</sup> Here I am presuming that this expected variability of income and its timing have been used in the computation of the taxable retirement income stream.

can be preserved by borrowing money while working and then paying that back with the retirement funds.

### 3.6 Policy Implications

At first glance, the use of retirement accounts in this paper may strike some as either a tax giveaway to households or an increased drain on tax revenues that ultimately will cause increased marginal tax rates. From a policy perspective, however, the household behavior of this paper implies that the introduction and simultaneous existence of traditional and Roth accounts improves the horizontal equity of the current American progressive tax code. Without the ability to time taxable income, people with identical average incomes but differing variance of income will pay different lifetime taxes under progressive income taxation. Once the decision is made to allow tax-deferred savings, such as traditional IRA and 401(k) accounts, the addition of Roth accounts improves the smoothing of taxable income across time and thus the correlation between lifetime income and lifetime taxes. A person who earns \$60,000 one year and \$70,000 the next can pay identical taxes across time to a person whose income is \$65,000 in each of the same years. But a person whose income averages \$70,000 will pay more on average, which is the guiding principle behind progressive taxation.

## 4. Conclusion

The results of this paper show that the rules of thumb for who should invest in traditional or Roth retirement accounts is only a starting point. For people with long time horizons and variable incomes, use of both types of accounts is necessary to maximize lifetime after-tax income, and this is done through equalizing marginal tax rates across time. Most people's lives are more complicated than the model presented in this paper, with variations in income uncertainty, lifespan uncertainty, tax rate uncertainty, bequest preferences, risk preferences and the like. However, the basic intuition of the result is clear and can serve as a benchmark for dealing with the above issues. Similarly, policy makers can see that giving households multiple types of tax-preferred retirement accounts

is a tax smoothing way to improve horizontal equity for households with income variability.

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## Appendix

Consider a three period model, with earnings in period 1 and 2 and retirement in period 3. Combine income earning periods 1 and 2, with the incomes, traditional and Roth account deposits indexed for the period that they occur. The analog to (7) now becomes:

$$\begin{aligned}
 (A1) \quad W(A_1, A_2) &= \int_{m_1=0}^{I_1-A_1} (1-bm_1) dm_1 + \int_{m_2=0}^{I_2-A_2} (1-bm_2) dm_2 \\
 &\quad - \int_{m_1=I_1-S_1}^{I_1-A_1} (1-bm_1) dm_1 - \int_{m_2=I_2-S_2}^{I_2-A_2} (1-bm_2) dm_2 \\
 &\quad + \int_{m_3=0}^{A_1(1+R)^2+A_2(1+R)} (1-bm_3) dm_3 + \int_{m_1=I_1-S_1}^{I_1-A_1} (1-bm_1) dm_1 (1+R)^2 \\
 &\quad + \int_{m_2=I_2-S_2}^{I_2-A_2} (1-bm_2) dm_2 (1+R).
 \end{aligned}$$

Taking the partial derivatives of W with respect to  $A_1$  and  $A_2$  and solving for them gives us:

$$\begin{aligned}
 (A2) \quad A_1^* &= \frac{(I_1 - I_2)(2 + R)}{R^2 + 3R + 3}. \\
 A_2^* &= \frac{(I_2 - I_1)(R^2 + 2R) + 2I_2 - I_1}{R^2 + 3R + 3}.
 \end{aligned}$$

When  $A_1^*$  and  $A_2^*$  are substituted into the taxable income expressions for periods 1, 2 and 3, which are the first, third and fifth expressions in (A1), the taxable incomes and marginal tax rates are equalized. Thus, maximizing the multiple earnings periods wealth function leads to the same central result of Proposition 1.